

# Mathematica 11.3 Integration Test Results

Test results for the 300 problems in "7.4.1 Inverse hyperbolic cotangent functions.m"

Problem 24: Result unnecessarily involves imaginary or complex numbers.

$$\int x^4 \operatorname{ArcCoth}[a x]^3 dx$$

Optimal (type 4, 196 leaves, 22 steps):

$$\frac{x^2}{20 a^3} + \frac{9 x \operatorname{ArcCoth}[a x]}{10 a^4} + \frac{x^3 \operatorname{ArcCoth}[a x]}{10 a^2} - \frac{9 \operatorname{ArcCoth}[a x]^2}{20 a^5} + \frac{3 x^2 \operatorname{ArcCoth}[a x]^2}{10 a^3} + \frac{3 x^4 \operatorname{ArcCoth}[a x]^2}{20 a} + \frac{\operatorname{ArcCoth}[a x]^3}{5 a^5} + \frac{1}{5} x^5 \operatorname{ArcCoth}[a x]^3 - \frac{3 \operatorname{ArcCoth}[a x]^2 \operatorname{Log}\left[\frac{2}{1-a x}\right]}{5 a^5} + \frac{\operatorname{Log}\left[1-a^2 x^2\right]}{2 a^5} - \frac{3 \operatorname{ArcCoth}[a x] \operatorname{PolyLog}\left[2, 1-\frac{2}{1-a x}\right]}{5 a^5} + \frac{3 \operatorname{PolyLog}\left[3, 1-\frac{2}{1-a x}\right]}{10 a^5}$$

Result (type 4, 175 leaves):

$$\frac{1}{40 a^5} \left( -2 - i \pi^3 + 2 a^2 x^2 + 36 a x \operatorname{ArcCoth}[a x] + 4 a^3 x^3 \operatorname{ArcCoth}[a x] - 18 \operatorname{ArcCoth}[a x]^2 + 12 a^2 x^2 \operatorname{ArcCoth}[a x]^2 + 6 a^4 x^4 \operatorname{ArcCoth}[a x]^2 + 8 \operatorname{ArcCoth}[a x]^3 + 8 a^5 x^5 \operatorname{ArcCoth}[a x]^3 - 24 \operatorname{ArcCoth}[a x]^2 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcCoth}[a x]}\right] - 40 \operatorname{Log}\left[\frac{1}{a \sqrt{1 - \frac{1}{a^2 x^2}}}\right] - 24 \operatorname{ArcCoth}[a x] \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcCoth}[a x]}\right] + 12 \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcCoth}[a x]}\right] \right)$$

Problem 26: Result unnecessarily involves imaginary or complex numbers.

$$\int x^2 \operatorname{ArcCoth}[a x]^3 dx$$

Optimal (type 4, 149 leaves, 11 steps):

$$\frac{x \operatorname{ArcCoth}[a x]}{a^2} - \frac{\operatorname{ArcCoth}[a x]^2}{2 a^3} + \frac{x^2 \operatorname{ArcCoth}[a x]^2}{2 a} +$$

$$\frac{\operatorname{ArcCoth}[a x]^3}{3 a^3} + \frac{1}{3} x^3 \operatorname{ArcCoth}[a x]^3 - \frac{\operatorname{ArcCoth}[a x]^2 \operatorname{Log}\left[\frac{2}{1-a x}\right]}{a^3} +$$

$$\frac{\operatorname{Log}\left[1-a^2 x^2\right]}{2 a^3} - \frac{\operatorname{ArcCoth}[a x] \operatorname{PolyLog}\left[2, 1-\frac{2}{1-a x}\right]}{a^3} + \frac{\operatorname{PolyLog}\left[3, 1-\frac{2}{1-a x}\right]}{2 a^3}$$

Result (type 4, 140 leaves):

$$\frac{1}{24 a^3} \left( -i \pi^3 + 24 a x \operatorname{ArcCoth}[a x] - 12 \operatorname{ArcCoth}[a x]^2 + 12 a^2 x^2 \operatorname{ArcCoth}[a x]^2 + 8 \operatorname{ArcCoth}[a x]^3 + \right.$$

$$8 a^3 x^3 \operatorname{ArcCoth}[a x]^3 - 24 \operatorname{ArcCoth}[a x]^2 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcCoth}[a x]}\right] - 24 \operatorname{Log}\left[\frac{1}{a \sqrt{1 - \frac{1}{a^2 x^2}}}\right] -$$

$$\left. 24 \operatorname{ArcCoth}[a x] \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcCoth}[a x]}\right] + 12 \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcCoth}[a x]}\right] \right)$$

**Problem 34: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{ArcCoth}[c x]^2}{d + e x} dx$$

Optimal (type 4, 164 leaves, 1 step):

$$-\frac{\operatorname{ArcCoth}[c x]^2 \operatorname{Log}\left[\frac{2}{1+c x}\right]}{e} + \frac{\operatorname{ArcCoth}[c x]^2 \operatorname{Log}\left[\frac{2 c(d+e x)}{(c d+e)(1+c x)}\right]}{e} + \frac{\operatorname{ArcCoth}[c x] \operatorname{PolyLog}\left[2, 1-\frac{2}{1+c x}\right]}{e} -$$

$$\frac{\operatorname{ArcCoth}[c x] \operatorname{PolyLog}\left[2, 1-\frac{2 c(d+e x)}{(c d+e)(1+c x)}\right]}{e} + \frac{\operatorname{PolyLog}\left[3, 1-\frac{2}{1+c x}\right]}{2 e} - \frac{\operatorname{PolyLog}\left[3, 1-\frac{2 c(d+e x)}{(c d+e)(1+c x)}\right]}{2 e}$$

Result (type 4, 741 leaves):

$$\begin{aligned}
 & \frac{1}{24 e^2} \left( -i e \pi^3 + 8 c d \operatorname{ArcCoth}[c x]^3 + 8 e \operatorname{ArcCoth}[c x]^3 - \right. \\
 & 24 e \operatorname{ArcCoth}[c x]^2 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcCoth}[c x]}\right] - 24 e \operatorname{ArcCoth}[c x] \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcCoth}[c x]}\right] + \\
 & 12 e \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcCoth}[c x]}\right] + \frac{1}{6 c^2 d^2 - 6 e^2} 24 (-c d + e) (c d + e) \\
 & \left. \left( -2 c d \operatorname{ArcCoth}[c x]^3 + 6 e \operatorname{ArcCoth}[c x]^3 + 4 c d \sqrt{1 - \frac{e^2}{c^2 d^2}} e^{-\operatorname{ArcTanh}\left[\frac{e}{c d}\right]} \operatorname{ArcCoth}[c x]^3 + \right. \right. \\
 & 6 i e \pi \operatorname{ArcCoth}[c x] \operatorname{Log}\left[\frac{1}{2} \left(e^{-\operatorname{ArcCoth}[c x]} + e^{\operatorname{ArcCoth}[c x]}\right)\right] + 6 e \operatorname{ArcCoth}[c x]^2 \\
 & \operatorname{Log}\left[1 + \frac{(c d + e) e^{2 \operatorname{ArcCoth}[c x]}}{-c d + e}\right] - 6 e \operatorname{ArcCoth}[c x]^2 \operatorname{Log}\left[1 - e^{\operatorname{ArcCoth}[c x] + \operatorname{ArcTanh}\left[\frac{e}{c d}\right]}\right] - \\
 & 6 e \operatorname{ArcCoth}[c x]^2 \operatorname{Log}\left[1 + e^{\operatorname{ArcCoth}[c x] + \operatorname{ArcTanh}\left[\frac{e}{c d}\right]}\right] - \\
 & 6 e \operatorname{ArcCoth}[c x]^2 \operatorname{Log}\left[1 - e^{2 \left(\operatorname{ArcCoth}[c x] + \operatorname{ArcTanh}\left[\frac{e}{c d}\right]\right)}\right] - 12 e \operatorname{ArcCoth}[c x] \\
 & \operatorname{ArcTanh}\left[\frac{e}{c d}\right] \operatorname{Log}\left[\frac{1}{2} i e^{-\operatorname{ArcCoth}[c x] - \operatorname{ArcTanh}\left[\frac{e}{c d}\right]} \left(-1 + e^{2 \left(\operatorname{ArcCoth}[c x] + \operatorname{ArcTanh}\left[\frac{e}{c d}\right]\right)}\right)\right] - \\
 & 6 e \operatorname{ArcCoth}[c x]^2 \operatorname{Log}\left[\frac{1}{2} e^{-\operatorname{ArcCoth}[c x]} (c d (-1 + e^{2 \operatorname{ArcCoth}[c x]}) + e (1 + e^{2 \operatorname{ArcCoth}[c x]}))\right] - \\
 & 6 i e \pi \operatorname{ArcCoth}[c x] \operatorname{Log}\left[\frac{1}{\sqrt{1 - \frac{1}{c^2 x^2}}}\right] + 6 e \operatorname{ArcCoth}[c x]^2 \operatorname{Log}\left[\frac{d + e x}{\sqrt{1 - \frac{1}{c^2 x^2}} x}\right] + \\
 & 12 e \operatorname{ArcCoth}[c x] \operatorname{ArcTanh}\left[\frac{e}{c d}\right] \operatorname{Log}\left[i \operatorname{Sinh}\left[\operatorname{ArcCoth}[c x] + \operatorname{ArcTanh}\left[\frac{e}{c d}\right]\right]\right] + \\
 & 6 e \operatorname{ArcCoth}[c x] \operatorname{PolyLog}\left[2, \frac{(c d + e) e^{2 \operatorname{ArcCoth}[c x]}}{c d - e}\right] - 12 e \operatorname{ArcCoth}[c x] \\
 & \operatorname{PolyLog}\left[2, -e^{\operatorname{ArcCoth}[c x] + \operatorname{ArcTanh}\left[\frac{e}{c d}\right]}\right] - 12 e \operatorname{ArcCoth}[c x] \operatorname{PolyLog}\left[2, e^{\operatorname{ArcCoth}[c x] + \operatorname{ArcTanh}\left[\frac{e}{c d}\right]}\right] - \\
 & 6 e \operatorname{ArcCoth}[c x] \operatorname{PolyLog}\left[2, e^{2 \left(\operatorname{ArcCoth}[c x] + \operatorname{ArcTanh}\left[\frac{e}{c d}\right]\right)}\right] - \\
 & 3 e \operatorname{PolyLog}\left[3, \frac{(c d + e) e^{2 \operatorname{ArcCoth}[c x]}}{c d - e}\right] + 12 e \operatorname{PolyLog}\left[3, -e^{\operatorname{ArcCoth}[c x] + \operatorname{ArcTanh}\left[\frac{e}{c d}\right]}\right] + \\
 & \left. \left. 12 e \operatorname{PolyLog}\left[3, e^{\operatorname{ArcCoth}[c x] + \operatorname{ArcTanh}\left[\frac{e}{c d}\right]}\right] + 3 e \operatorname{PolyLog}\left[3, e^{2 \left(\operatorname{ArcCoth}[c x] + \operatorname{ArcTanh}\left[\frac{e}{c d}\right]\right)}\right]\right) \right)
 \end{aligned}$$

### Problem 41: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcCoth}[a x]}{(c + d x^2)^3} dx$$

Optimal (type 4, 657 leaves, 23 steps):

$$\begin{aligned} & \frac{a}{8 c (a^2 c + d) (c + d x^2)} + \frac{x \text{ArcCoth}[a x]}{4 c (c + d x^2)^2} + \frac{3 x \text{ArcCoth}[a x]}{8 c^2 (c + d x^2)} + \frac{3 \text{ArcCoth}[a x] \text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]}{8 c^{5/2} \sqrt{d}} + \\ & \frac{3 i \text{Log}\left[\frac{\sqrt{d} (1 - a x)}{i a \sqrt{c} + \sqrt{d}}\right] \text{Log}\left[1 - \frac{i \sqrt{d} x}{\sqrt{c}}\right]}{32 c^{5/2} \sqrt{d}} - \frac{3 i \text{Log}\left[-\frac{\sqrt{d} (1 + a x)}{i a \sqrt{c} - \sqrt{d}}\right] \text{Log}\left[1 - \frac{i \sqrt{d} x}{\sqrt{c}}\right]}{32 c^{5/2} \sqrt{d}} - \\ & \frac{3 i \text{Log}\left[-\frac{\sqrt{d} (1 - a x)}{i a \sqrt{c} - \sqrt{d}}\right] \text{Log}\left[1 + \frac{i \sqrt{d} x}{\sqrt{c}}\right]}{32 c^{5/2} \sqrt{d}} + \frac{3 i \text{Log}\left[\frac{\sqrt{d} (1 + a x)}{i a \sqrt{c} + \sqrt{d}}\right] \text{Log}\left[1 + \frac{i \sqrt{d} x}{\sqrt{c}}\right]}{32 c^{5/2} \sqrt{d}} + \\ & \frac{a (5 a^2 c + 3 d) \text{Log}[1 - a^2 x^2]}{16 c^2 (a^2 c + d)^2} - \frac{a (5 a^2 c + 3 d) \text{Log}[c + d x^2]}{16 c^2 (a^2 c + d)^2} + \frac{3 i \text{PolyLog}\left[2, \frac{a (\sqrt{c} - i \sqrt{d} x)}{a \sqrt{c} - i \sqrt{d}}\right]}{32 c^{5/2} \sqrt{d}} - \\ & \frac{3 i \text{PolyLog}\left[2, \frac{a (\sqrt{c} - i \sqrt{d} x)}{a \sqrt{c} + i \sqrt{d}}\right]}{32 c^{5/2} \sqrt{d}} + \frac{3 i \text{PolyLog}\left[2, \frac{a (\sqrt{c} + i \sqrt{d} x)}{a \sqrt{c} - i \sqrt{d}}\right]}{32 c^{5/2} \sqrt{d}} - \frac{3 i \text{PolyLog}\left[2, \frac{a (\sqrt{c} + i \sqrt{d} x)}{a \sqrt{c} + i \sqrt{d}}\right]}{32 c^{5/2} \sqrt{d}} \end{aligned}$$

Result (type 4, 1838 leaves):

$$\begin{aligned} & a^5 \left( -\frac{5 \text{Log}\left[1 + \frac{(a^2 c + d) \text{Cosh}[2 \text{ArcCoth}[a x]]}{-a^2 c + d}\right]}{16 a^2 c (a^2 c + d)^2} - \frac{3 d \text{Log}\left[1 + \frac{(a^2 c + d) \text{Cosh}[2 \text{ArcCoth}[a x]]}{-a^2 c + d}\right]}{16 a^4 c^2 (a^2 c + d)^2} \right) + \\ & \frac{1}{32 a^2 c \sqrt{a^2 c d} (a^2 c + d)} 3 \left( -2 i \text{ArcCos}\left[-\frac{-a^2 c + d}{a^2 c + d}\right] \text{ArcTan}\left[\frac{a c}{\sqrt{a^2 c d} x}\right] + \right. \\ & \left. 4 \text{ArcCoth}[a x] \text{ArcTan}\left[\frac{a d x}{\sqrt{a^2 c d}}\right] - \left( \text{ArcCos}\left[-\frac{-a^2 c + d}{a^2 c + d}\right] - 2 \text{ArcTan}\left[\frac{a c}{\sqrt{a^2 c d} x}\right] \right) \right. \\ & \left. \text{Log}\left[1 - \frac{(-a^2 c + d - 2 i \sqrt{a^2 c d}) \left(2 d - \frac{2 i \sqrt{a^2 c d}}{a x}\right)}{(a^2 c + d) \left(2 d + \frac{2 i \sqrt{a^2 c d}}{a x}\right)}\right] + \left( -\text{ArcCos}\left[-\frac{-a^2 c + d}{a^2 c + d}\right] - \right. \right. \\ & \left. \left. 2 \text{ArcTan}\left[\frac{a c}{\sqrt{a^2 c d} x}\right] \right) \text{Log}\left[1 - \frac{(-a^2 c + d + 2 i \sqrt{a^2 c d}) \left(2 d - \frac{2 i \sqrt{a^2 c d}}{a x}\right)}{(a^2 c + d) \left(2 d + \frac{2 i \sqrt{a^2 c d}}{a x}\right)}\right] + \right) \end{aligned}$$

$$\begin{aligned}
 & \left( \operatorname{ArcCos} \left[ -\frac{-a^2 c + d}{a^2 c + d} \right] + 2 i \left( -i \operatorname{ArcTan} \left[ \frac{a c}{\sqrt{a^2 c d} x} \right] - i \operatorname{ArcTan} \left[ \frac{a d x}{\sqrt{a^2 c d}} \right] \right) \right) \\
 & \operatorname{Log} \left[ \frac{\sqrt{2} \sqrt{a^2 c d} e^{-\operatorname{ArcCoth}[a x]}}{\sqrt{a^2 c + d} \sqrt{-a^2 c + d + (a^2 c + d) \operatorname{Cosh}[2 \operatorname{ArcCoth}[a x]]}} \right] + \\
 & \left( \operatorname{ArcCos} \left[ -\frac{-a^2 c + d}{a^2 c + d} \right] - 2 i \left( -i \operatorname{ArcTan} \left[ \frac{a c}{\sqrt{a^2 c d} x} \right] - i \operatorname{ArcTan} \left[ \frac{a d x}{\sqrt{a^2 c d}} \right] \right) \right) \\
 & \operatorname{Log} \left[ \frac{\sqrt{2} \sqrt{a^2 c d} e^{\operatorname{ArcCoth}[a x]}}{\sqrt{a^2 c + d} \sqrt{-a^2 c + d + (a^2 c + d) \operatorname{Cosh}[2 \operatorname{ArcCoth}[a x]]}} \right] + \\
 & i \left( \operatorname{PolyLog} \left[ 2, \frac{(-a^2 c + d - 2 i \sqrt{a^2 c d}) \left( 2 d - \frac{2 i \sqrt{a^2 c d}}{a x} \right)}{(a^2 c + d) \left( 2 d + \frac{2 i \sqrt{a^2 c d}}{a x} \right)} \right] - \right. \\
 & \left. \operatorname{PolyLog} \left[ 2, \frac{(-a^2 c + d + 2 i \sqrt{a^2 c d}) \left( 2 d - \frac{2 i \sqrt{a^2 c d}}{a x} \right)}{(a^2 c + d) \left( 2 d + \frac{2 i \sqrt{a^2 c d}}{a x} \right)} \right] \right) + \\
 & \frac{1}{32 a^4 c^2 \sqrt{a^2 c d} (a^2 c + d)} 3 d \left( -2 i \operatorname{ArcCos} \left[ -\frac{-a^2 c + d}{a^2 c + d} \right] \operatorname{ArcTan} \left[ \frac{a c}{\sqrt{a^2 c d} x} \right] + \right. \\
 & 4 \operatorname{ArcCoth}[a x] \operatorname{ArcTan} \left[ \frac{a d x}{\sqrt{a^2 c d}} \right] - \left( \operatorname{ArcCos} \left[ -\frac{-a^2 c + d}{a^2 c + d} \right] - 2 \operatorname{ArcTan} \left[ \frac{a c}{\sqrt{a^2 c d} x} \right] \right) \\
 & \operatorname{Log} \left[ 1 - \frac{(-a^2 c + d - 2 i \sqrt{a^2 c d}) \left( 2 d - \frac{2 i \sqrt{a^2 c d}}{a x} \right)}{(a^2 c + d) \left( 2 d + \frac{2 i \sqrt{a^2 c d}}{a x} \right)} \right] + \\
 & \left( -\operatorname{ArcCos} \left[ -\frac{-a^2 c + d}{a^2 c + d} \right] - 2 \operatorname{ArcTan} \left[ \frac{a c}{\sqrt{a^2 c d} x} \right] \right) \\
 & \operatorname{Log} \left[ 1 - \frac{(-a^2 c + d + 2 i \sqrt{a^2 c d}) \left( 2 d - \frac{2 i \sqrt{a^2 c d}}{a x} \right)}{(a^2 c + d) \left( 2 d + \frac{2 i \sqrt{a^2 c d}}{a x} \right)} \right] + \\
 & \left( \operatorname{ArcCos} \left[ -\frac{-a^2 c + d}{a^2 c + d} \right] + 2 i \left( -i \operatorname{ArcTan} \left[ \frac{a c}{\sqrt{a^2 c d} x} \right] - i \operatorname{ArcTan} \left[ \frac{a d x}{\sqrt{a^2 c d}} \right] \right) \right) \\
 & \operatorname{Log} \left[ \frac{\sqrt{2} \sqrt{a^2 c d} e^{-\operatorname{ArcCoth}[a x]}}{\sqrt{a^2 c + d} \sqrt{-a^2 c + d + (a^2 c + d) \operatorname{Cosh}[2 \operatorname{ArcCoth}[a x]]}} \right] +
 \end{aligned}$$

$$\left( \text{ArcCos} \left[ -\frac{-a^2 c + d}{a^2 c + d} \right] - 2 i \left( -i \text{ArcTan} \left[ \frac{a c}{\sqrt{a^2 c d} x} \right] - i \text{ArcTan} \left[ \frac{a d x}{\sqrt{a^2 c d}} \right] \right) \right) \left( \text{Log} \left[ \frac{\sqrt{2} \sqrt{a^2 c d} e^{\text{ArcCoth}[a x]}}{\sqrt{a^2 c + d} \sqrt{-a^2 c + d + (a^2 c + d) \text{Cosh}[2 \text{ArcCoth}[a x]]}} \right] + i \left( \text{PolyLog} \left[ 2, \frac{(-a^2 c + d - 2 i \sqrt{a^2 c d}) \left( 2 d - \frac{2 i \sqrt{a^2 c d}}{a x} \right)}{(a^2 c + d) \left( 2 d + \frac{2 i \sqrt{a^2 c d}}{a x} \right)} \right] - \text{PolyLog} \left[ 2, \frac{(-a^2 c + d + 2 i \sqrt{a^2 c d}) \left( 2 d - \frac{2 i \sqrt{a^2 c d}}{a x} \right)}{(a^2 c + d) \left( 2 d + \frac{2 i \sqrt{a^2 c d}}{a x} \right)} \right] \right) \right) \left( \frac{(d \text{ArcCoth}[a x] \text{Sinh}[2 \text{ArcCoth}[a x]])}{(2 a^2 c (a^2 c + d) (-a^2 c + d + a^2 c \text{Cosh}[2 \text{ArcCoth}[a x]] + d \text{Cosh}[2 \text{ArcCoth}[a x]])^2)} - \frac{(2 a^2 c d - 5 a^4 c^2 \text{ArcCoth}[a x] \text{Sinh}[2 \text{ArcCoth}[a x]] - 8 a^2 c d \text{ArcCoth}[a x] \text{Sinh}[2 \text{ArcCoth}[a x]] - 3 d^2 \text{ArcCoth}[a x] \text{Sinh}[2 \text{ArcCoth}[a x]])}{(8 a^4 c^2 (a^2 c + d)^2 (-a^2 c + d + a^2 c \text{Cosh}[2 \text{ArcCoth}[a x]] + d \text{Cosh}[2 \text{ArcCoth}[a x]])} \right)$$

**Problem 66: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{ArcCoth}[a + b x]}{x} dx$$

Optimal (type 4, 92 leaves, 5 steps):

$$-\text{ArcCoth}[a + b x] \text{Log} \left[ \frac{2}{1 + a + b x} \right] + \text{ArcCoth}[a + b x] \text{Log} \left[ \frac{2 b x}{(1 - a) (1 + a + b x)} \right] + \frac{1}{2} \text{PolyLog} \left[ 2, 1 - \frac{2}{1 + a + b x} \right] - \frac{1}{2} \text{PolyLog} \left[ 2, 1 - \frac{2 b x}{(1 - a) (1 + a + b x)} \right]$$

Result (type 4, 259 leaves):

$$\begin{aligned}
 & (\text{ArcCoth}[a + b x] - \text{ArcTanh}[a + b x]) \text{Log}[x] + \\
 & \text{ArcTanh}[a + b x] \left( -\text{Log}\left[\frac{1}{\sqrt{1 - (a + b x)^2}}\right] + \text{Log}[-i \text{Sinh}[\text{ArcTanh}[a] - \text{ArcTanh}[a + b x]]] \right) + \\
 & \frac{1}{8} \left( 4 (\text{ArcTanh}[a] - \text{ArcTanh}[a + b x])^2 - (\pi - 2 i \text{ArcTanh}[a + b x])^2 - \right. \\
 & \quad 8 (\text{ArcTanh}[a] - \text{ArcTanh}[a + b x]) \text{Log}\left[1 - e^{2 \text{ArcTanh}[a] - 2 \text{ArcTanh}[a + b x]}\right] - \\
 & \quad 4 i (\pi - 2 i \text{ArcTanh}[a + b x]) \text{Log}\left[1 + e^{2 \text{ArcTanh}[a + b x]}\right] + \\
 & \quad 4 (i \pi + 2 \text{ArcTanh}[a + b x]) \text{Log}\left[\frac{2}{\sqrt{1 - (a + b x)^2}}\right] + \\
 & \quad 8 (\text{ArcTanh}[a] - \text{ArcTanh}[a + b x]) \text{Log}[-2 i \text{Sinh}[\text{ArcTanh}[a] - \text{ArcTanh}[a + b x]]] - \\
 & \quad \left. 4 \text{PolyLog}\left[2, e^{2 \text{ArcTanh}[a] - 2 \text{ArcTanh}[a + b x]}\right] - 4 \text{PolyLog}\left[2, -e^{2 \text{ArcTanh}[a + b x]}\right] \right)
 \end{aligned}$$

Problem 70: Result more than twice size of optimal antiderivative.

$$\int x^2 \text{ArcCoth}[a + b x]^2 dx$$

Optimal (type 4, 204 leaves, 15 steps):

$$\begin{aligned}
 & \frac{x}{3 b^2} - \frac{2 a (a + b x) \text{ArcCoth}[a + b x]}{b^3} + \frac{(a + b x)^2 \text{ArcCoth}[a + b x]}{3 b^3} + \frac{a (3 + a^2) \text{ArcCoth}[a + b x]^2}{3 b^3} + \\
 & \frac{(1 + 3 a^2) \text{ArcCoth}[a + b x]^2}{3 b^3} + \frac{1}{3} x^3 \text{ArcCoth}[a + b x]^2 - \frac{\text{ArcTanh}[a + b x]}{3 b^3} - \\
 & \frac{2 (1 + 3 a^2) \text{ArcCoth}[a + b x] \text{Log}\left[\frac{2}{1 - a - b x}\right]}{3 b^3} - \frac{a \text{Log}\left[1 - (a + b x)^2\right]}{b^3} - \frac{(1 + 3 a^2) \text{PolyLog}\left[2, -\frac{1 + a + b x}{1 - a - b x}\right]}{3 b^3}
 \end{aligned}$$

Result (type 4, 615 leaves):

$$\begin{aligned}
 & -\frac{1}{12 b^3} (a + b x) \sqrt{1 - \frac{1}{(a + b x)^2}} \left(1 - (a + b x)^2\right) \left( \frac{4 \operatorname{ArcCoth}[a + b x]}{(a + b x) \sqrt{1 - \frac{1}{(a + b x)^2}}} + \right. \\
 & \frac{3 \operatorname{ArcCoth}[a + b x]^2}{(a + b x) \sqrt{1 - \frac{1}{(a + b x)^2}}} - \frac{12 a \operatorname{ArcCoth}[a + b x]^2}{(a + b x) \sqrt{1 - \frac{1}{(a + b x)^2}}} + \frac{9 a^2 \operatorname{ArcCoth}[a + b x]^2}{(a + b x) \sqrt{1 - \frac{1}{(a + b x)^2}}} + \\
 & \frac{1}{\sqrt{1 - \frac{1}{(a + b x)^2}}} \left(-1 + 6 a \operatorname{ArcCoth}[a + b x] + 3 \operatorname{ArcCoth}[a + b x]^2 - 3 a^2 \operatorname{ArcCoth}[a + b x]^2\right) + \\
 & \operatorname{Cosh}[3 \operatorname{ArcCoth}[a + b x]] - 6 a \operatorname{ArcCoth}[a + b x] \operatorname{Cosh}[3 \operatorname{ArcCoth}[a + b x]] + \\
 & \operatorname{ArcCoth}[a + b x]^2 \operatorname{Cosh}[3 \operatorname{ArcCoth}[a + b x]] + 3 a^2 \operatorname{ArcCoth}[a + b x]^2 \operatorname{Cosh}[3 \operatorname{ArcCoth}[a + b x]] + \\
 & \frac{6 \operatorname{ArcCoth}[a + b x] \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcCoth}[a + b x]}\right]}{(a + b x) \sqrt{1 - \frac{1}{(a + b x)^2}}} + \frac{18 a^2 \operatorname{ArcCoth}[a + b x] \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcCoth}[a + b x]}\right]}{(a + b x) \sqrt{1 - \frac{1}{(a + b x)^2}}} - \\
 & \frac{18 a \operatorname{Log}\left[\frac{1}{(a + b x) \sqrt{1 - \frac{1}{(a + b x)^2}}}\right]}{(a + b x) \sqrt{1 - \frac{1}{(a + b x)^2}}} + \frac{4 (1 + 3 a^2) \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcCoth}[a + b x]}\right]}{(a + b x)^3 \left(1 - \frac{1}{(a + b x)^2}\right)^{3/2}} - \\
 & \operatorname{ArcCoth}[a + b x]^2 \operatorname{Sinh}[3 \operatorname{ArcCoth}[a + b x]] - 3 a^2 \operatorname{ArcCoth}[a + b x]^2 \operatorname{Sinh}[3 \operatorname{ArcCoth}[a + b x]] - \\
 & 2 \operatorname{ArcCoth}[a + b x] \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcCoth}[a + b x]}\right] \operatorname{Sinh}[3 \operatorname{ArcCoth}[a + b x]] - \\
 & 6 a^2 \operatorname{ArcCoth}[a + b x] \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcCoth}[a + b x]}\right] \operatorname{Sinh}[3 \operatorname{ArcCoth}[a + b x]] + \\
 & \left. 6 a \operatorname{Log}\left[\frac{1}{(a + b x) \sqrt{1 - \frac{1}{(a + b x)^2}}}\right] \operatorname{Sinh}[3 \operatorname{ArcCoth}[a + b x]] \right)
 \end{aligned}$$

**Problem 73: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{ArcCoth}[a + b x]^2}{x} dx$$

Optimal (type 4, 148 leaves, 2 steps):



$$\begin{aligned}
 & -\text{ArcCoth}[a + b x]^2 \text{Log}\left[\frac{2}{1 + a + b x}\right] + \text{ArcCoth}[a + b x]^2 \text{Log}\left[\frac{2 b x}{(1 - a)(1 + a + b x)}\right] + \\
 & \text{ArcCoth}[a + b x] \text{PolyLog}\left[2, 1 - \frac{2}{1 + a + b x}\right] - \text{ArcCoth}[a + b x] \text{PolyLog}\left[2, 1 - \frac{2 b x}{(1 - a)(1 + a + b x)}\right] + \\
 & \frac{1}{2} \text{PolyLog}\left[3, 1 - \frac{2}{1 + a + b x}\right] - \frac{1}{2} \text{PolyLog}\left[3, 1 - \frac{2 b x}{(1 - a)(1 + a + b x)}\right]
 \end{aligned}$$

Result (type 4, 675 leaves):

$$\begin{aligned}
 & -\frac{i \pi^3}{24} - \frac{2}{3} \text{ArcCoth}[a + b x]^3 - \frac{2}{3} a \text{ArcCoth}[a + b x]^3 + \frac{2}{3} \sqrt{1 - \frac{1}{a^2}} a e^{\text{ArcTanh}\left[\frac{1}{a}\right]} \text{ArcCoth}[a + b x]^3 - \\
 & i \pi \text{ArcCoth}[a + b x] \text{Log}\left[\frac{1}{2} \left(e^{-\text{ArcCoth}[a + b x]} + e^{\text{ArcCoth}[a + b x]}\right)\right] - \\
 & \text{ArcCoth}[a + b x]^2 \text{Log}\left[1 - e^{2 \text{ArcCoth}[a + b x]}\right] - \text{ArcCoth}[a + b x]^2 \text{Log}\left[1 - \frac{(-1 + a) e^{2 \text{ArcCoth}[a + b x]}}{1 + a}\right] + \\
 & \text{ArcCoth}[a + b x]^2 \text{Log}\left[1 - e^{2 \text{ArcCoth}[a + b x] - 2 \text{ArcTanh}\left[\frac{1}{a}\right]}\right] + \\
 & \text{ArcCoth}[a + b x]^2 \text{Log}\left[1 - e^{\text{ArcCoth}[a + b x] - \text{ArcTanh}\left[\frac{1}{a}\right]}\right] + \\
 & \text{ArcCoth}[a + b x]^2 \text{Log}\left[1 + e^{\text{ArcCoth}[a + b x] - \text{ArcTanh}\left[\frac{1}{a}\right]}\right] - \\
 & 2 \text{ArcCoth}[a + b x] \text{ArcTanh}\left[\frac{1}{a}\right] \text{Log}\left[\frac{1}{2} i \left(e^{\text{ArcCoth}[a + b x] - \text{ArcTanh}\left[\frac{1}{a}\right]} - e^{-\text{ArcCoth}[a + b x] + \text{ArcTanh}\left[\frac{1}{a}\right]}\right)\right] + \\
 & \text{ArcCoth}[a + b x]^2 \text{Log}\left[\frac{1}{2} e^{-\text{ArcCoth}[a + b x]} \left(-1 - e^{2 \text{ArcCoth}[a + b x]} + a \left(-1 + e^{2 \text{ArcCoth}[a + b x]}\right)\right)\right] + \\
 & i \pi \text{ArcCoth}[a + b x] \text{Log}\left[\frac{1}{\sqrt{1 - \frac{1}{(a + b x)^2}}}\right] - \text{ArcCoth}[a + b x]^2 \text{Log}\left[-\frac{b x}{(a + b x) \sqrt{1 - \frac{1}{(a + b x)^2}}}\right] + \\
 & 2 \text{ArcCoth}[a + b x] \text{ArcTanh}\left[\frac{1}{a}\right] \text{Log}\left[i \text{Sinh}\left[\text{ArcCoth}[a + b x] - \text{ArcTanh}\left[\frac{1}{a}\right]\right]\right] - \\
 & \text{ArcCoth}[a + b x] \text{PolyLog}\left[2, e^{2 \text{ArcCoth}[a + b x]}\right] - \\
 & \text{ArcCoth}[a + b x] \text{PolyLog}\left[2, \frac{(-1 + a) e^{2 \text{ArcCoth}[a + b x]}}{1 + a}\right] + \\
 & \text{ArcCoth}[a + b x] \text{PolyLog}\left[2, e^{2 \text{ArcCoth}[a + b x] - 2 \text{ArcTanh}\left[\frac{1}{a}\right]}\right] + \\
 & 2 \text{ArcCoth}[a + b x] \text{PolyLog}\left[2, -e^{\text{ArcCoth}[a + b x] - \text{ArcTanh}\left[\frac{1}{a}\right]}\right] + \\
 & 2 \text{ArcCoth}[a + b x] \text{PolyLog}\left[2, e^{\text{ArcCoth}[a + b x] - \text{ArcTanh}\left[\frac{1}{a}\right]}\right] + \frac{1}{2} \text{PolyLog}\left[3, e^{2 \text{ArcCoth}[a + b x]}\right] + \\
 & \frac{1}{2} \text{PolyLog}\left[3, \frac{(-1 + a) e^{2 \text{ArcCoth}[a + b x]}}{1 + a}\right] - \frac{1}{2} \text{PolyLog}\left[3, e^{2 \text{ArcCoth}[a + b x] - 2 \text{ArcTanh}\left[\frac{1}{a}\right]}\right] - \\
 & 2 \text{PolyLog}\left[3, -e^{\text{ArcCoth}[a + b x] - \text{ArcTanh}\left[\frac{1}{a}\right]}\right] - 2 \text{PolyLog}\left[3, e^{\text{ArcCoth}[a + b x] - \text{ArcTanh}\left[\frac{1}{a}\right]}\right]
 \end{aligned}$$

**Problem 74: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\text{ArcCoth}[a + b x]^2}{x^2} dx$$

Optimal (type 4, 251 leaves, 17 steps):

$$\begin{aligned}
 & -\frac{\text{ArcCoth}[a + b x]^2}{x} + \frac{b \text{ArcCoth}[a + b x] \text{Log}\left[\frac{2}{1-a-b x}\right]}{1-a} + \\
 & \frac{b \text{ArcCoth}[a + b x] \text{Log}\left[\frac{2}{1+a+b x}\right]}{1+a} - \frac{2 b \text{ArcCoth}[a + b x] \text{Log}\left[\frac{2}{1+a+b x}\right]}{1-a^2} + \\
 & \frac{2 b \text{ArcCoth}[a + b x] \text{Log}\left[\frac{2 b x}{(1-a)(1+a+b x)}\right]}{1-a^2} + \frac{b \text{PolyLog}\left[2, -\frac{1+a+b x}{1-a-b x}\right]}{2(1-a)} - \\
 & \frac{b \text{PolyLog}\left[2, 1-\frac{2}{1+a+b x}\right]}{2(1+a)} + \frac{b \text{PolyLog}\left[2, 1-\frac{2}{1+a+b x}\right]}{1-a^2} - \frac{b \text{PolyLog}\left[2, 1-\frac{2 b x}{(1-a)(1+a+b x)}\right]}{1-a^2}
 \end{aligned}$$

Result (type 4, 206 leaves):

$$\begin{aligned}
 & \frac{1}{(-1+a^2)x} \left( -\left( -1+a^2 + \sqrt{1-\frac{1}{a^2}} a b e^{\text{ArcTanh}\left[\frac{1}{a}\right] x} \right) \text{ArcCoth}[a + b x]^2 + \right. \\
 & b x \text{ArcCoth}[a + b x] \left( -i \pi + 2 \text{ArcTanh}\left[\frac{1}{a}\right] - 2 \text{Log}\left[1 - e^{-2 \text{ArcCoth}[a + b x] + 2 \text{ArcTanh}\left[\frac{1}{a}\right]}\right] \right) + \\
 & b x \left( i \pi \left( \text{Log}\left[1 + e^{2 \text{ArcCoth}[a + b x]}\right] - \text{Log}\left[\frac{1}{\sqrt{1-\frac{1}{(a+b x)^2}}}\right] \right) + 2 \text{ArcTanh}\left[\frac{1}{a}\right] \right. \\
 & \left. \left( \text{Log}\left[1 - e^{-2 \text{ArcCoth}[a + b x] + 2 \text{ArcTanh}\left[\frac{1}{a}\right]}\right] - \text{Log}\left[i \text{Sinh}\left[\text{ArcCoth}[a + b x] - \text{ArcTanh}\left[\frac{1}{a}\right]\right]\right] \right) \right) + \\
 & \left. b x \text{PolyLog}\left[2, e^{-2 \text{ArcCoth}[a + b x] + 2 \text{ArcTanh}\left[\frac{1}{a}\right]}\right] \right)
 \end{aligned}$$

**Problem 75: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\text{ArcCoth}[a + b x]^2}{x^3} dx$$

Optimal (type 4, 370 leaves, 21 steps):

$$\begin{aligned}
 & - \frac{b \operatorname{ArcCoth}[a + b x]}{(1 - a^2) x} - \frac{\operatorname{ArcCoth}[a + b x]^2}{2 x^2} + \frac{b^2 \operatorname{Log}[x]}{(1 - a^2)^2} + \frac{b^2 \operatorname{ArcCoth}[a + b x] \operatorname{Log}\left[\frac{2}{1 - a - b x}\right]}{2 (1 - a)^2} - \\
 & \frac{b^2 \operatorname{Log}[1 - a - b x]}{2 (1 - a)^2 (1 + a)} - \frac{b^2 \operatorname{ArcCoth}[a + b x] \operatorname{Log}\left[\frac{2}{1 + a + b x}\right]}{2 (1 + a)^2} - \frac{2 a b^2 \operatorname{ArcCoth}[a + b x] \operatorname{Log}\left[\frac{2}{1 + a + b x}\right]}{(1 - a^2)^2} + \\
 & \frac{2 a b^2 \operatorname{ArcCoth}[a + b x] \operatorname{Log}\left[\frac{2 b x}{(1 - a)(1 + a + b x)}\right]}{(1 - a^2)^2} - \frac{b^2 \operatorname{Log}[1 + a + b x]}{2 (1 - a)(1 + a)^2} + \frac{b^2 \operatorname{PolyLog}\left[2, -\frac{1 + a + b x}{1 - a - b x}\right]}{4 (1 - a)^2} + \\
 & \frac{b^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + a + b x}\right]}{4 (1 + a)^2} + \frac{a b^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + a + b x}\right]}{(1 - a^2)^2} - \frac{a b^2 \operatorname{PolyLog}\left[2, 1 - \frac{2 b x}{(1 - a)(1 + a + b x)}\right]}{(1 - a^2)^2}
 \end{aligned}$$

Result (type 4, 291 leaves):

$$\begin{aligned}
 & \frac{1}{2 (-1 + a^2)^2 x^2} \left( \left( -1 - a^4 + b^2 x^2 + a^2 \left( 2 + b^2 \left( -1 + 2 \sqrt{1 - \frac{1}{a^2}} e^{\operatorname{ArcTanh}\left[\frac{1}{a}\right]} x^2 \right) \right) \right) \operatorname{ArcCoth}[a + b x]^2 + \right. \\
 & 2 b x \operatorname{ArcCoth}[a + b x] \\
 & \left. \left( -1 + a^2 + a b x + i a b \pi x - 2 a b x \operatorname{ArcTanh}\left[\frac{1}{a}\right] + 2 a b x \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcCoth}[a + b x] + 2 \operatorname{ArcTanh}\left[\frac{1}{a}\right]}\right] \right) \right) + \\
 & 2 b^2 x^2 \left( -i a \pi \operatorname{Log}\left[1 + e^{2 \operatorname{ArcCoth}[a + b x]}\right] + i a \pi \operatorname{Log}\left[\frac{1}{\sqrt{1 - \frac{1}{(a + b x)^2}}}\right] + \right. \\
 & \left. \operatorname{Log}\left[-\frac{b x}{(a + b x) \sqrt{1 - \frac{1}{(a + b x)^2}}}\right] - 2 a \operatorname{ArcTanh}\left[\frac{1}{a}\right] \right. \\
 & \left. \left( \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcCoth}[a + b x] + 2 \operatorname{ArcTanh}\left[\frac{1}{a}\right]}\right] - \operatorname{Log}\left[i \operatorname{Sinh}\left[\operatorname{ArcCoth}[a + b x] - \operatorname{ArcTanh}\left[\frac{1}{a}\right]\right]\right] \right) \right) - \\
 & \left. 2 a b^2 x^2 \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcCoth}[a + b x] + 2 \operatorname{ArcTanh}\left[\frac{1}{a}\right]}\right] \right)
 \end{aligned}$$

**Problem 76: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{ArcCoth}[a + b x]}{c + d x^2} dx$$

Optimal (type 4, 673 leaves, 15 steps):

$$\frac{\text{Log}\left[-\frac{1-a-bx}{a+bx}\right] \text{Log}\left[1 + \frac{(b^2 c+a^2 d)(1-a-bx)}{(b^2 c-b\sqrt{-c}\sqrt{d}-(1-a)ad)(a+bx)}\right]}{4\sqrt{-c}\sqrt{d}} - \frac{\text{Log}\left[-\frac{1-a-bx}{a+bx}\right] \text{Log}\left[1 + \frac{(b^2 c+a^2 d)(1-a-bx)}{(b^2 c+b\sqrt{-c}\sqrt{d}-(1-a)ad)(a+bx)}\right]}{4\sqrt{-c}\sqrt{d}} + \frac{\text{Log}\left[\frac{1+a+bx}{a+bx}\right] \text{Log}\left[1 - \frac{(b^2 c+a^2 d)(1+a+bx)}{(b^2 c-b\sqrt{-c}\sqrt{d}+a(1+a)d)(a+bx)}\right]}{4\sqrt{-c}\sqrt{d}} - \frac{\text{Log}\left[\frac{1+a+bx}{a+bx}\right] \text{Log}\left[1 - \frac{(b^2 c+a^2 d)(1+a+bx)}{(b^2 c+b\sqrt{-c}\sqrt{d}+a(1+a)d)(a+bx)}\right]}{4\sqrt{-c}\sqrt{d}} + \frac{\text{PolyLog}\left[2, -\frac{(b^2 c+a^2 d)(1-a-bx)}{(b^2 c-b\sqrt{-c}\sqrt{d}-(1-a)ad)(a+bx)}\right]}{4\sqrt{-c}\sqrt{d}} - \frac{\text{PolyLog}\left[2, -\frac{(b^2 c+a^2 d)(1-a-bx)}{(b^2 c+b\sqrt{-c}\sqrt{d}-(1-a)ad)(a+bx)}\right]}{4\sqrt{-c}\sqrt{d}} + \frac{\text{PolyLog}\left[2, \frac{(b^2 c+a^2 d)(1+a+bx)}{(b^2 c-b\sqrt{-c}\sqrt{d}+a(1+a)d)(a+bx)}\right]}{4\sqrt{-c}\sqrt{d}} - \frac{\text{PolyLog}\left[2, \frac{(b^2 c+a^2 d)(1+a+bx)}{(b^2 c+b\sqrt{-c}\sqrt{d}+a(1+a)d)(a+bx)}\right]}{4\sqrt{-c}\sqrt{d}}$$

Result (type 4, 1450 leaves):

$$-\frac{1}{4(1-a^2)\sqrt{c}d(a+bx)^2\left(1-\frac{1}{(a+bx)^2}\right)} \left(1-(a+bx)^2\right) \left(-4(-1+a^2)\sqrt{d}\text{ArcCoth}[a+bx]\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right] + 2i\sqrt{d}\text{ArcTan}\left[\frac{(-1+a)\sqrt{d}}{b\sqrt{c}}\right]\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right] - 2i\sqrt{d}a^2\sqrt{d}\text{ArcTan}\left[\frac{(-1+a)\sqrt{d}}{b\sqrt{c}}\right]\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right] - 2i\sqrt{d}\text{ArcTan}\left[\frac{(1+a)\sqrt{d}}{b\sqrt{c}}\right]\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right] + 2i\sqrt{d}a^2\sqrt{d}\text{ArcTan}\left[\frac{(1+a)\sqrt{d}}{b\sqrt{c}}\right]\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right] - 2b\sqrt{c}\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right]^2 + b\sqrt{c}\sqrt{\frac{b^2c+(-1+a)^2d}{b^2c}}e^{-i\text{ArcTan}\left[\frac{(-1+a)\sqrt{d}}{b\sqrt{c}}\right]}\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right]^2 + \dots$$

$$\begin{aligned}
 & a b \sqrt{c} \sqrt{\frac{b^2 c + (-1+a)^2 d}{b^2 c}} e^{-i \operatorname{ArcTan}\left[\frac{(-1+a)\sqrt{d}}{b\sqrt{c}}\right]} \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]^2 + \\
 & b \sqrt{c} \sqrt{\frac{b^2 c + (1+a)^2 d}{b^2 c}} e^{-i \operatorname{ArcTan}\left[\frac{(1+a)\sqrt{d}}{b\sqrt{c}}\right]} \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]^2 - \\
 & a b \sqrt{c} \sqrt{\frac{b^2 c + (1+a)^2 d}{b^2 c}} e^{-i \operatorname{ArcTan}\left[\frac{(1+a)\sqrt{d}}{b\sqrt{c}}\right]} \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]^2 + \\
 & 2 \sqrt{d} \operatorname{ArcTan}\left[\frac{(-1+a)\sqrt{d}}{b\sqrt{c}}\right] \operatorname{Log}\left[1 - e^{-2i\left(\operatorname{ArcTan}\left[\frac{(-1+a)\sqrt{d}}{b\sqrt{c}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]\right)}\right] - \\
 & 2 a^2 \sqrt{d} \operatorname{ArcTan}\left[\frac{(-1+a)\sqrt{d}}{b\sqrt{c}}\right] \operatorname{Log}\left[1 - e^{-2i\left(\operatorname{ArcTan}\left[\frac{(-1+a)\sqrt{d}}{b\sqrt{c}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]\right)}\right] + \\
 & 2 \sqrt{d} \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right] \operatorname{Log}\left[1 - e^{-2i\left(\operatorname{ArcTan}\left[\frac{(-1+a)\sqrt{d}}{b\sqrt{c}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]\right)}\right] - \\
 & 2 a^2 \sqrt{d} \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right] \operatorname{Log}\left[1 - e^{-2i\left(\operatorname{ArcTan}\left[\frac{(-1+a)\sqrt{d}}{b\sqrt{c}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]\right)}\right] - \\
 & 2 \sqrt{d} \operatorname{ArcTan}\left[\frac{(1+a)\sqrt{d}}{b\sqrt{c}}\right] \operatorname{Log}\left[1 - e^{-2i\left(\operatorname{ArcTan}\left[\frac{(1+a)\sqrt{d}}{b\sqrt{c}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]\right)}\right] + \\
 & 2 a^2 \sqrt{d} \operatorname{ArcTan}\left[\frac{(1+a)\sqrt{d}}{b\sqrt{c}}\right] \operatorname{Log}\left[1 - e^{-2i\left(\operatorname{ArcTan}\left[\frac{(1+a)\sqrt{d}}{b\sqrt{c}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]\right)}\right] - \\
 & 2 \sqrt{d} \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right] \operatorname{Log}\left[1 - e^{-2i\left(\operatorname{ArcTan}\left[\frac{(1+a)\sqrt{d}}{b\sqrt{c}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]\right)}\right] + \\
 & 2 a^2 \sqrt{d} \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right] \operatorname{Log}\left[1 - e^{-2i\left(\operatorname{ArcTan}\left[\frac{(1+a)\sqrt{d}}{b\sqrt{c}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]\right)}\right] - \\
 & 2 \sqrt{d} \operatorname{ArcTan}\left[\frac{(-1+a)\sqrt{d}}{b\sqrt{c}}\right] \operatorname{Log}\left[-\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{(-1+a)\sqrt{d}}{b\sqrt{c}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]\right]\right] + \\
 & 2 a^2 \sqrt{d} \operatorname{ArcTan}\left[\frac{(-1+a)\sqrt{d}}{b\sqrt{c}}\right] \operatorname{Log}\left[-\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{(-1+a)\sqrt{d}}{b\sqrt{c}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]\right]\right] + \\
 & 2 \sqrt{d} \operatorname{ArcTan}\left[\frac{(1+a)\sqrt{d}}{b\sqrt{c}}\right] \operatorname{Log}\left[-\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{(1+a)\sqrt{d}}{b\sqrt{c}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]\right]\right] - \\
 & 2 a^2 \sqrt{d} \operatorname{ArcTan}\left[\frac{(1+a)\sqrt{d}}{b\sqrt{c}}\right] \operatorname{Log}\left[-\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{(1+a)\sqrt{d}}{b\sqrt{c}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]\right]\right] - \\
 & i(-1+a^2)\sqrt{d} \operatorname{PolyLog}\left[2, e^{-2i\left(\operatorname{ArcTan}\left[\frac{(-1+a)\sqrt{d}}{b\sqrt{c}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]\right)}\right] +
 \end{aligned}$$

$$i (-1 + a^2) \sqrt{d} \text{PolyLog}\left[2, e^{-2i \left(\text{ArcTan}\left[\frac{(1+a)\sqrt{d}}{b\sqrt{c}}\right] + \text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right]\right)}\right]\right)$$

**Problem 77: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{ArcCoth}[a + b x]}{c + d x} dx$$

Optimal (type 4, 120 leaves, 5 steps):

$$-\frac{\text{ArcCoth}[a + b x] \text{Log}\left[\frac{2}{1+a+bx}\right]}{d} + \frac{\text{ArcCoth}[a + b x] \text{Log}\left[\frac{2b(c+dx)}{(b c+d-a d)(1+a+bx)}\right]}{d} + \frac{\text{PolyLog}\left[2, 1 - \frac{2}{1+a+bx}\right]}{2d} - \frac{\text{PolyLog}\left[2, 1 - \frac{2b(c+dx)}{(b c+d-a d)(1+a+bx)}\right]}{2d}$$

Result (type 4, 330 leaves):

$$\frac{1}{d} \left( (\text{ArcCoth}[a + b x] - \text{ArcTanh}[a + b x]) \text{Log}[c + d x] + \text{ArcTanh}[a + b x] \left( -\text{Log}\left[\frac{1}{\sqrt{1 - (a + b x)^2}}\right] + \text{Log}\left[i \text{Sinh}\left[\text{ArcTanh}\left[\frac{b c - a d}{d}\right] + \text{ArcTanh}[a + b x]\right]\right] \right) + \frac{1}{8} \left( -(\pi - 2i \text{ArcTanh}[a + b x])^2 + 4 \left(\text{ArcTanh}\left[\frac{b c - a d}{d}\right] + \text{ArcTanh}[a + b x]\right)^2 - 4i(\pi - 2i \text{ArcTanh}[a + b x]) \text{Log}\left[1 + e^{2 \text{ArcTanh}[a + b x]}\right] + 8 \left(\text{ArcTanh}\left[\frac{b c - a d}{d}\right] + \text{ArcTanh}[a + b x]\right) \text{Log}\left[1 - e^{-2 \left(\text{ArcTanh}\left[\frac{b c - a d}{d}\right] + \text{ArcTanh}[a + b x]\right)}\right] + 4(i\pi + 2 \text{ArcTanh}[a + b x]) \text{Log}\left[\frac{2}{\sqrt{1 - (a + b x)^2}}\right] - 8 \left(\text{ArcTanh}\left[\frac{b c - a d}{d}\right] + \text{ArcTanh}[a + b x]\right) \text{Log}\left[2i \text{Sinh}\left[\text{ArcTanh}\left[\frac{b c - a d}{d}\right] + \text{ArcTanh}[a + b x]\right]\right] - 4 \text{PolyLog}\left[2, -e^{2 \text{ArcTanh}[a + b x]}\right] - 4 \text{PolyLog}\left[2, e^{-2 \left(\text{ArcTanh}\left[\frac{b c - a d}{d}\right] + \text{ArcTanh}[a + b x]\right)}\right] \right) \right)$$

**Problem 78: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\text{ArcCoth}[a + b x]}{c + \frac{d}{x}} dx$$

Optimal (type 4, 292 leaves, 37 steps):

$$\begin{aligned}
 & \frac{(1-a-bx) \operatorname{Log}\left[-\frac{1-a-bx}{a+bx}\right]}{2bc} + \frac{\operatorname{Log}[a+bx]}{2bc} + \frac{\operatorname{Log}[1+a+bx]}{2bc} + \frac{(a+bx) \operatorname{Log}\left[\frac{1+a+bx}{a+bx}\right]}{2bc} - \\
 & \frac{d \operatorname{Log}\left[\frac{c(1-a-bx)}{c-ac+bd}\right] \operatorname{Log}[d+cx]}{2c^2} + \frac{d \operatorname{Log}\left[-\frac{1-a-bx}{a+bx}\right] \operatorname{Log}[d+cx]}{2c^2} + \frac{d \operatorname{Log}\left[\frac{c(1+a+bx)}{c+ac-bd}\right] \operatorname{Log}[d+cx]}{2c^2} - \\
 & \frac{d \operatorname{Log}\left[\frac{1+a+bx}{a+bx}\right] \operatorname{Log}[d+cx]}{2c^2} + \frac{d \operatorname{PolyLog}\left[2, -\frac{b(d+cx)}{c+ac-bd}\right]}{2c^2} - \frac{d \operatorname{PolyLog}\left[2, \frac{b(d+cx)}{c+ac+bd}\right]}{2c^2}
 \end{aligned}$$

Result (type 4, 502 leaves):

$$\begin{aligned}
 & \frac{1}{2bc^3} \left( 2ac^2 \operatorname{ArcCoth}[a+bx] - i bcd \pi \operatorname{ArcCoth}[a+bx] + \right. \\
 & 2bc^2 x \operatorname{ArcCoth}[a+bx] + bcd \operatorname{ArcCoth}[a+bx]^2 + abc d \operatorname{ArcCoth}[a+bx]^2 - \\
 & b^2 d^2 \operatorname{ArcCoth}[a+bx]^2 - abcd \sqrt{1 - \frac{c^2}{(ac-bd)^2}} e^{\operatorname{ArcTanh}\left[\frac{c}{ac-bd}\right]} \operatorname{ArcCoth}[a+bx]^2 + \\
 & b^2 d^2 \sqrt{1 - \frac{c^2}{(ac-bd)^2}} e^{\operatorname{ArcTanh}\left[\frac{c}{ac-bd}\right]} \operatorname{ArcCoth}[a+bx]^2 + \\
 & 2bcd \operatorname{ArcCoth}[a+bx] \operatorname{ArcTanh}\left[\frac{c}{ac-bd}\right] + 2bcd \operatorname{ArcCoth}[a+bx] \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcCoth}[a+bx]}\right] + \\
 & i bcd \pi \operatorname{Log}\left[1 + e^{2 \operatorname{ArcCoth}[a+bx]}\right] - 2bcd \operatorname{ArcCoth}[a+bx] \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcCoth}[a+bx] + 2 \operatorname{ArcTanh}\left[\frac{c}{ac-bd}\right]}\right] + \\
 & 2bcd \operatorname{ArcTanh}\left[\frac{c}{ac-bd}\right] \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcCoth}[a+bx] + 2 \operatorname{ArcTanh}\left[\frac{c}{ac-bd}\right]}\right] - \\
 & i bcd \pi \operatorname{Log}\left[\frac{1}{\sqrt{1 - \frac{1}{(a+bx)^2}}}\right] - 2c^2 \operatorname{Log}\left[\frac{1}{(a+bx) \sqrt{1 - \frac{1}{(a+bx)^2}}}\right] - \\
 & 2bcd \operatorname{ArcTanh}\left[\frac{c}{ac-bd}\right] \operatorname{Log}\left[i \operatorname{Sinh}\left[\operatorname{ArcCoth}[a+bx] - \operatorname{ArcTanh}\left[\frac{c}{ac-bd}\right]\right]\right] - \\
 & \left. bcd \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcCoth}[a+bx]}\right] + bcd \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcCoth}[a+bx] + 2 \operatorname{ArcTanh}\left[\frac{c}{ac-bd}\right]}\right] \right)
 \end{aligned}$$

**Problem 79:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{ArcCoth}[a+bx]}{c + \frac{d}{x^2}} dx$$

Optimal (type 4, 738 leaves, 57 steps):

$$\begin{aligned}
 & \frac{(1-a-bx) \operatorname{Log}[-1+a+bx]}{2bc} + \frac{x \left( \operatorname{Log}[-1+a+bx] - \operatorname{Log}\left[-\frac{1-a-bx}{a+bx}\right] - \operatorname{Log}[a+bx] \right)}{2c} - \\
 & \frac{\sqrt{d} \operatorname{ArcTan}\left[\frac{\sqrt{c}x}{\sqrt{d}}\right] \left( \operatorname{Log}[-1+a+bx] - \operatorname{Log}\left[-\frac{1-a-bx}{a+bx}\right] - \operatorname{Log}[a+bx] \right)}{2c^{3/2}} + \\
 & \frac{(1+a+bx) \operatorname{Log}[1+a+bx]}{2bc} + \frac{x \left( \operatorname{Log}[a+bx] - \operatorname{Log}[1+a+bx] + \operatorname{Log}\left[\frac{1+a+bx}{a+bx}\right] \right)}{2c} - \\
 & \frac{\sqrt{d} \operatorname{ArcTan}\left[\frac{\sqrt{c}x}{\sqrt{d}}\right] \left( \operatorname{Log}[a+bx] - \operatorname{Log}[1+a+bx] + \operatorname{Log}\left[\frac{1+a+bx}{a+bx}\right] \right)}{2c^{3/2}} + \\
 & \frac{\sqrt{d} \operatorname{Log}[-1+a+bx] \operatorname{Log}\left[-\frac{b(\sqrt{d}-\sqrt{c}x)}{(1-a)\sqrt{c}-b\sqrt{d}}\right]}{4(-c)^{3/2}} - \frac{\sqrt{d} \operatorname{Log}[1+a+bx] \operatorname{Log}\left[\frac{b(\sqrt{d}-\sqrt{c}x)}{(1+a)\sqrt{c}+b\sqrt{d}}\right]}{4(-c)^{3/2}} + \\
 & \frac{\sqrt{d} \operatorname{Log}[1+a+bx] \operatorname{Log}\left[-\frac{b(\sqrt{d}+\sqrt{c}x)}{(1+a)\sqrt{c}-b\sqrt{d}}\right]}{4(-c)^{3/2}} - \frac{\sqrt{d} \operatorname{Log}[-1+a+bx] \operatorname{Log}\left[\frac{b(\sqrt{d}+\sqrt{c}x)}{(1-a)\sqrt{c}+b\sqrt{d}}\right]}{4(-c)^{3/2}} + \\
 & \frac{\sqrt{d} \operatorname{PolyLog}\left[2, \frac{\sqrt{c}(1-a-bx)}{(1-a)\sqrt{c}-b\sqrt{d}}\right]}{4(-c)^{3/2}} - \frac{\sqrt{d} \operatorname{PolyLog}\left[2, \frac{\sqrt{c}(1-a-bx)}{(1-a)\sqrt{c}+b\sqrt{d}}\right]}{4(-c)^{3/2}} + \\
 & \frac{\sqrt{d} \operatorname{PolyLog}\left[2, \frac{\sqrt{c}(1+a+bx)}{(1+a)\sqrt{c}-b\sqrt{d}}\right]}{4(-c)^{3/2}} - \frac{\sqrt{d} \operatorname{PolyLog}\left[2, \frac{\sqrt{c}(1+a+bx)}{(1+a)\sqrt{c}+b\sqrt{d}}\right]}{4(-c)^{3/2}}
 \end{aligned}$$

Result (type 4, 15460 leaves):

$$\begin{aligned}
 & -\frac{1}{(a+bx)^2 \left(1 - \frac{1}{(a+bx)^2}\right)} \left(1 - (a+bx)^2\right) \left( \frac{(a+bx) \operatorname{ArcCoth}[a+bx] - \operatorname{Log}\left[\frac{1}{(a+bx)\sqrt{1 - \frac{1}{(a+bx)^2}}}\right]}{bc} + \right. \\
 & \frac{1}{c} 2bd \left( \frac{\operatorname{ArcCoth}[a+bx] \operatorname{ArcTan}\left[\frac{-a\sqrt{c} + \frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]}{2b\sqrt{c}\sqrt{d}} + \frac{1}{2(a^2c+b^2d)\left(-1 + \frac{1}{(a+bx)^2}\right)} \right. \\
 & \left. \left. \left(-1 + \frac{c\left(a\sqrt{c} - b\sqrt{d}\left(\frac{a\sqrt{c}}{b\sqrt{d}} - \frac{a^2c+b^2d}{b\sqrt{c}\sqrt{d}(a+bx)}\right)\right)^2}{(a^2c+b^2d)^2}\right) \right) \left( -\frac{(a^2c+b^2d)^2 \operatorname{ArcTan}\left[\frac{a\sqrt{c} - \frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]^2}{2(a^4c^2+b^4d^2 - a^2c(c-2b^2d))} + \right.
 \end{aligned}$$



$$\begin{aligned}
 & \frac{1}{2} a^2 \sqrt{c} \left( \frac{\sqrt{c} e^{i \operatorname{ArcTan}\left[\frac{-a c+a^2 c+b^2 d}{b \sqrt{c} \sqrt{d}}\right]} \operatorname{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b \sqrt{c} \sqrt{d}}\right]^2}{(-a c+a^2 c+b^2 d) \sqrt{1+\frac{(-a c+a^2 c+b^2 d)^2}{b^2 c d}}} + \frac{1}{b \sqrt{d} \left(1+\frac{(-a c+a^2 c+b^2 d)^2}{b^2 c d}\right)} \right. \\
 & \left. \left( -\pi \operatorname{Log}\left[1+e^{-2 i \operatorname{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b \sqrt{c} \sqrt{d}}\right]}\right] - i \operatorname{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b \sqrt{c} \sqrt{d}}\right] \left( \pi - 2 \operatorname{ArcTan}\left[\frac{-a c+a^2 c+b^2 d}{b \sqrt{c} \sqrt{d}}\right] - 2 i \operatorname{Log}\left[1-e^{2 i \left(\operatorname{ArcTan}\left[\frac{-a c+a^2 c+b^2 d}{b \sqrt{c} \sqrt{d}}\right]+\operatorname{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b \sqrt{c} \sqrt{d}}\right]}\right)}\right] \right) - \right. \\
 & \left. 2 \operatorname{ArcTan}\left[\frac{-a c+a^2 c+b^2 d}{b \sqrt{c} \sqrt{d}}\right] \operatorname{Log}\left[1-e^{2 i \left(\operatorname{ArcTan}\left[\frac{-a c+a^2 c+b^2 d}{b \sqrt{c} \sqrt{d}}\right]+\operatorname{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b \sqrt{c} \sqrt{d}}\right]}\right)}\right] + \right. \\
 & \left. \pi \operatorname{Log}\left[\frac{1}{\sqrt{\frac{(a^2 c+b^2 d)\left(c+\frac{a^2 c-b^2 d}{(a+b x)^2}-\frac{2 a c}{a+b x}\right)}{b^2 c d}}}\right] + 2 \operatorname{ArcTan}\left[\frac{-a c+a^2 c+b^2 d}{b \sqrt{c} \sqrt{d}}\right] \right. \\
 & \left. \operatorname{Log}\left[\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{-a c+a^2 c+b^2 d}{b \sqrt{c} \sqrt{d}}\right]\right] + \operatorname{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b \sqrt{c} \sqrt{d}}\right]\right] \right] + \\
 & \left. i \operatorname{PolyLog}\left[2, e^{2 i \left(\operatorname{ArcTan}\left[\frac{-a c+a^2 c+b^2 d}{b \sqrt{c} \sqrt{d}}\right]+\operatorname{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b \sqrt{c} \sqrt{d}}\right]}\right)}\right] \right) \right) - \\
 & \frac{1}{(-a c+a^2 c+b^2 d) \sqrt{1+\frac{(-a c+a^2 c+b^2 d)^2}{b^2 c d}}} a^3 c \left( e^{i \operatorname{ArcTan}\left[\frac{-a c+a^2 c+b^2 d}{b \sqrt{c} \sqrt{d}}\right]} \right. \\
 & \left. \operatorname{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b \sqrt{c} \sqrt{d}}\right]^2 + \frac{1}{b \sqrt{c} \sqrt{d} \sqrt{1+\frac{(-a c+a^2 c+b^2 d)^2}{b^2 c d}}} (-a c+a^2 c+b^2 d) \right)
 \end{aligned}$$

$$\left( -\pi \operatorname{Log}\left[1 + e^{-2i \operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]}\right] - i \operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \right) \left( \pi - 2 \operatorname{ArcTan}\left[\frac{-ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] - 2i \operatorname{Log}\left[1 - e^{2i \left(\operatorname{ArcTan}\left[\frac{-ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] + \operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]\right)}\right] \right) -$$

$$2 \operatorname{ArcTan}\left[\frac{-ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \operatorname{Log}\left[1 - e^{2i \left(\operatorname{ArcTan}\left[\frac{-ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] + \operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]\right)}\right] +$$

$$\pi \operatorname{Log}\left[\frac{1}{\sqrt{\frac{(a^2c + b^2d)\left(c + \frac{a^2c + b^2d}{(a+bx)^2} - \frac{2ac}{a+bx}\right)}{b^2cd}}}\right] + 2 \operatorname{ArcTan}\left[\frac{-ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]$$

$$\operatorname{Log}\left[\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{-ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]\right] + \operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]\right] +$$

$$i \operatorname{PolyLog}\left[2, e^{2i \left(\operatorname{ArcTan}\left[\frac{-ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] + \operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]\right)}\right] \Bigg) +$$

$$\frac{1}{4(-ac + a^2c + b^2d)\sqrt{1 + \frac{(-ac + a^2c + b^2d)^2}{b^2cd}}} 3a^4c \left( e^{i \operatorname{ArcTan}\left[\frac{-ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]} \right.$$

$$\operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]^2 + \frac{1}{b\sqrt{c}\sqrt{d}\sqrt{1 + \frac{(-ac + a^2c + b^2d)^2}{b^2cd}}} (-ac + a^2c + b^2d)$$

$$\left. -\pi \operatorname{Log}\left[1 + e^{-2i \operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]}\right] - i \operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \right) \left( \pi - 2 \operatorname{ArcTan}\left[\frac{-ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] - 2i \operatorname{Log}\left[1 - e^{2i \left(\operatorname{ArcTan}\left[\frac{-ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] + \operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]\right)}\right] \right) -$$

$$\begin{aligned}
 & \left. \frac{-a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] - 2 i \operatorname{Log} \left[ 1 - e^{2 i \left( \operatorname{ArcTan} \left[ \frac{-a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] + \operatorname{ArcTan} \left[ \frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] \right)} \right] \right) - \\
 & 2 \operatorname{ArcTan} \left[ \frac{-a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \operatorname{Log} \left[ 1 - e^{2 i \left( \operatorname{ArcTan} \left[ \frac{-a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] + \operatorname{ArcTan} \left[ \frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] \right)} \right] + \\
 & \pi \operatorname{Log} \left[ \frac{1}{\sqrt{\frac{(a^2 c + b^2 d) \left( c + \frac{a^2 c + b^2 d}{(a + b x)^2} - \frac{2 a c}{a + b x} \right)}{b^2 c d}}} \right] + 2 \operatorname{ArcTan} \left[ \frac{-a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \\
 & \operatorname{Log} \left[ \operatorname{Sin} \left[ \operatorname{ArcTan} \left[ \frac{-a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \right] + \operatorname{ArcTan} \left[ \frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] \right] + \\
 & \left. i \operatorname{PolyLog} \left[ 2, e^{2 i \left( \operatorname{ArcTan} \left[ \frac{-a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] + \operatorname{ArcTan} \left[ \frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] \right)} \right] \right) \right) + \\
 & \frac{1}{4 b^2 d (-a c + a^2 c + b^2 d) \sqrt{1 + \frac{(-a c + a^2 c + b^2 d)^2}{b^2 c d}}} a^4 c^2 \left( e^{i \operatorname{ArcTan} \left[ \frac{-a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right]} \right. \\
 & \operatorname{ArcTan} \left[ \frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right]^2 + \frac{1}{b \sqrt{c} \sqrt{d} \sqrt{1 + \frac{(-a c + a^2 c + b^2 d)^2}{b^2 c d}}} (-a c + a^2 c + b^2 d) \\
 & \left( -\pi \operatorname{Log} \left[ 1 + e^{-2 i \operatorname{ArcTan} \left[ \frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right]} \right] - i \operatorname{ArcTan} \left[ \frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] \left( \pi - 2 \operatorname{ArcTan} \left[ \right. \right. \right. \\
 & \left. \left. \left. \frac{-a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] - 2 i \operatorname{Log} \left[ 1 - e^{2 i \left( \operatorname{ArcTan} \left[ \frac{-a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] + \operatorname{ArcTan} \left[ \frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] \right)} \right] \right) \right) - \\
 & 2 \operatorname{ArcTan} \left[ \frac{-a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \operatorname{Log} \left[ 1 - e^{2 i \left( \operatorname{ArcTan} \left[ \frac{-a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] + \operatorname{ArcTan} \left[ \frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] \right)} \right] +
 \end{aligned}$$

$$\begin{aligned}
 & \pi \operatorname{Log} \left[ \frac{1}{\sqrt{\frac{(a^2 c + b^2 d) \left( c + \frac{a^2 c + b^2 d}{(a + b x)^2} - \frac{2 a c}{a + b x} \right)}{b^2 c d}}} \right] + 2 \operatorname{ArcTan} \left[ \frac{-a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \\
 & \operatorname{Log} \left[ \operatorname{Sin} \left[ \operatorname{ArcTan} \left[ \frac{-a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \right] + \operatorname{ArcTan} \left[ \frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] \right] + \\
 & \left. \left. \left. i \operatorname{PolyLog} \left[ 2, e^{2 i \left( \operatorname{ArcTan} \left[ \frac{-a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] + \operatorname{ArcTan} \left[ \frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] \right)} \right] \right] \right] - \right. \\
 & \left. \frac{1}{2 b^2 d (-a c + a^2 c + b^2 d) \sqrt{1 + \frac{(-a c + a^2 c + b^2 d)^2}{b^2 c d}}} a^5 c^2 \left( e^{i \operatorname{ArcTan} \left[ \frac{-a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right]} \right. \right. \\
 & \left. \operatorname{ArcTan} \left[ \frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right]^2 + \frac{1}{b \sqrt{c} \sqrt{d} \sqrt{1 + \frac{(-a c + a^2 c + b^2 d)^2}{b^2 c d}}} (-a c + a^2 c + b^2 d) \right. \\
 & \left. \left( -\pi \operatorname{Log} \left[ 1 + e^{-2 i \operatorname{ArcTan} \left[ \frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right]} \right] - i \operatorname{ArcTan} \left[ \frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] \left( \pi - 2 \operatorname{ArcTan} \left[ \right. \right. \right. \\
 & \left. \left. \left. \frac{-a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] - 2 i \operatorname{Log} \left[ 1 - e^{2 i \left( \operatorname{ArcTan} \left[ \frac{-a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] + \operatorname{ArcTan} \left[ \frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] \right)} \right] \right) \right] - \right. \\
 & \left. 2 \operatorname{ArcTan} \left[ \frac{-a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \operatorname{Log} \left[ 1 - e^{2 i \left( \operatorname{ArcTan} \left[ \frac{-a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] + \operatorname{ArcTan} \left[ \frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] \right)} \right] \right] + \\
 & \left. \pi \operatorname{Log} \left[ \frac{1}{\sqrt{\frac{(a^2 c + b^2 d) \left( c + \frac{a^2 c + b^2 d}{(a + b x)^2} - \frac{2 a c}{a + b x} \right)}{b^2 c d}}} \right] + 2 \operatorname{ArcTan} \left[ \frac{-a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \right.
 \end{aligned}$$

$$\begin{aligned}
 & \text{Log}\left[\text{Sin}\left[\text{ArcTan}\left[\frac{-a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] + \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]\right]\right] + \\
 & \left. \left. \left. i \text{PolyLog}\left[2, e^{2 i \left(\text{ArcTan}\left[\frac{-a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] + \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]\right)}\right]\right] \right) \right) + \\
 & \frac{1}{4 b^2 d (-a c + a^2 c + b^2 d) \sqrt{1 + \frac{(-a c + a^2 c + b^2 d)^2}{b^2 c d}}} a^6 c^2 \left( e^{i \text{ArcTan}\left[\frac{-a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]} \right. \\
 & \left. \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]^2 + \frac{1}{b \sqrt{c} \sqrt{d} \sqrt{1 + \frac{(-a c + a^2 c + b^2 d)^2}{b^2 c d}}} (-a c + a^2 c + b^2 d) \right. \\
 & \left. \left( -\pi \text{Log}\left[1 + e^{-2 i \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]}\right] - i \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] \left( \pi - 2 \text{ArcTan}\left[\frac{-a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] - 2 i \text{Log}\left[1 - e^{2 i \left(\text{ArcTan}\left[\frac{-a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] + \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]\right)}\right] \right) \right) - \\
 & \left. 2 \text{ArcTan}\left[\frac{-a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] \text{Log}\left[1 - e^{2 i \left(\text{ArcTan}\left[\frac{-a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] + \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]\right)}\right] \right) + \\
 & \pi \text{Log}\left[\frac{1}{\sqrt{\frac{(a^2 c + b^2 d) \left(c + \frac{a^2 c + b^2 d}{(a + b x)^2} - \frac{2 a c}{a + b x}\right)}{b^2 c d}}}\right] + 2 \text{ArcTan}\left[\frac{-a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] \\
 & \left. \text{Log}\left[\text{Sin}\left[\text{ArcTan}\left[\frac{-a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] + \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]\right]\right] + \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left. \left. \left. \left. i \operatorname{PolyLog}\left[2, e^{2i \left( \operatorname{ArcTan}\left[\frac{-ac+a^2c+b^2d}{b\sqrt{c}\sqrt{d}}\right] + \operatorname{ArcTan}\left[\frac{ac-\frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]\right)}\right] \right) \right) \right) + \right. \\
 & \frac{1}{4(-ac+a^2c+b^2d)\sqrt{1+\frac{(-ac+a^2c+b^2d)^2}{b^2cd}}} b^2d \left( e^{i \operatorname{ArcTan}\left[\frac{-ac+a^2c+b^2d}{b\sqrt{c}\sqrt{d}}\right]} \right. \\
 & \operatorname{ArcTan}\left[\frac{ac-\frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]^2 + \frac{1}{b\sqrt{c}\sqrt{d}\sqrt{1+\frac{(-ac+a^2c+b^2d)^2}{b^2cd}}} (-ac+a^2c+b^2d) \\
 & \left. \left( -\pi \operatorname{Log}\left[1+e^{-2i \operatorname{ArcTan}\left[\frac{ac-\frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]} - i \operatorname{ArcTan}\left[\frac{ac-\frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]\right] \left( \pi - 2 \operatorname{ArcTan}\left[\frac{-ac+a^2c+b^2d}{b\sqrt{c}\sqrt{d}}\right] - 2i \operatorname{Log}\left[1-e^{2i \left( \operatorname{ArcTan}\left[\frac{-ac+a^2c+b^2d}{b\sqrt{c}\sqrt{d}}\right] + \operatorname{ArcTan}\left[\frac{ac-\frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]\right)}\right] \right) - \right. \\
 & \left. 2 \operatorname{ArcTan}\left[\frac{-ac+a^2c+b^2d}{b\sqrt{c}\sqrt{d}}\right] \operatorname{Log}\left[1-e^{2i \left( \operatorname{ArcTan}\left[\frac{-ac+a^2c+b^2d}{b\sqrt{c}\sqrt{d}}\right] + \operatorname{ArcTan}\left[\frac{ac-\frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]\right)}\right] \right) + \\
 & \pi \operatorname{Log}\left[\frac{1}{\sqrt{\frac{(a^2c+b^2d)\left(c+\frac{a^2c+b^2d}{(a+bx)^2}\frac{2ac}{a+bx}\right)}{b^2cd}}}\right] + 2 \operatorname{ArcTan}\left[\frac{-ac+a^2c+b^2d}{b\sqrt{c}\sqrt{d}}\right] \\
 & \left. \left. \left. \left. \operatorname{Log}\left[\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{-ac+a^2c+b^2d}{b\sqrt{c}\sqrt{d}}\right]\right] + \operatorname{ArcTan}\left[\frac{ac-\frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]\right] \right) \right) \right) + \right. \\
 & \left. \left. \left. \left. i \operatorname{PolyLog}\left[2, e^{2i \left( \operatorname{ArcTan}\left[\frac{-ac+a^2c+b^2d}{b\sqrt{c}\sqrt{d}}\right] + \operatorname{ArcTan}\left[\frac{ac-\frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]\right)}\right] \right) \right) \right) - \right.
 \end{aligned}$$

$$\begin{aligned}
 & \frac{1}{2(-ac + a^2c + b^2d) \sqrt{1 + \frac{(-ac + a^2c + b^2d)^2}{b^2cd}}} a b^2 d \left( e^{i \operatorname{ArcTan}\left[\frac{-ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]} \right. \\
 & \operatorname{ArcTan}\left[\frac{ac - \frac{a^2c + b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]^2 + \frac{1}{b\sqrt{c}\sqrt{d} \sqrt{1 + \frac{(-ac + a^2c + b^2d)^2}{b^2cd}}} (-ac + a^2c + b^2d) \\
 & \left. \left( -\pi \operatorname{Log}\left[1 + e^{-2i \operatorname{ArcTan}\left[\frac{ac - \frac{a^2c + b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]}\right] - i \operatorname{ArcTan}\left[\frac{ac - \frac{a^2c + b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] \left( \pi - 2 \operatorname{ArcTan}\left[\frac{-ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] - 2i \operatorname{Log}\left[1 - e^{2i \left(\operatorname{ArcTan}\left[\frac{-ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] + \operatorname{ArcTan}\left[\frac{ac - \frac{a^2c + b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]}\right)}\right] \right) - \right. \\
 & \left. 2 \operatorname{ArcTan}\left[\frac{-ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \operatorname{Log}\left[1 - e^{2i \left(\operatorname{ArcTan}\left[\frac{-ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] + \operatorname{ArcTan}\left[\frac{ac - \frac{a^2c + b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]}\right)}\right] \right) + \\
 & \pi \operatorname{Log}\left[\frac{1}{\sqrt{\frac{(a^2c + b^2d) \left(c + \frac{a^2c + b^2d}{(a+bx)^2} - \frac{2ac}{a+bx}\right)}{b^2cd}}}\right] + 2 \operatorname{ArcTan}\left[\frac{-ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \\
 & \operatorname{Log}\left[\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{-ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]\right] + \operatorname{ArcTan}\left[\frac{ac - \frac{a^2c + b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]\right] + \\
 & \left. i \operatorname{PolyLog}\left[2, e^{2i \left(\operatorname{ArcTan}\left[\frac{-ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] + \operatorname{ArcTan}\left[\frac{ac - \frac{a^2c + b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]}\right)}\right] \right) + \\
 & \frac{1}{4(-ac + a^2c + b^2d) \sqrt{1 + \frac{(-ac + a^2c + b^2d)^2}{b^2cd}}} 3 a^2 b^2 d \left( e^{i \operatorname{ArcTan}\left[\frac{-ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]} \right)
 \end{aligned}$$

$$\begin{aligned}
 & \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]^2 + \frac{1}{b \sqrt{c} \sqrt{d} \sqrt{1 + \frac{(-a c + a^2 c + b^2 d)^2}{b^2 c d}}} (-a c + a^2 c + b^2 d) \\
 & \left( -\pi \text{Log}\left[1 + e^{-2 i \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]}\right] - i \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] \left( \pi - 2 \text{ArcTan}\left[\frac{-a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] - 2 i \text{Log}\left[1 - e^{2 i \left(\text{ArcTan}\left[\frac{-a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] + \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]}\right)}\right] \right) - \\
 & 2 \text{ArcTan}\left[\frac{-a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] \text{Log}\left[1 - e^{2 i \left(\text{ArcTan}\left[\frac{-a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] + \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]}\right)}\right] + \\
 & \pi \text{Log}\left[\frac{1}{\sqrt{\frac{(a^2 c + b^2 d) \left(c + \frac{a^2 c + b^2 d}{(a + b x)^2} \frac{2 a c}{a + b x}\right)}{b^2 c d}}}\right] + 2 \text{ArcTan}\left[\frac{-a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] \\
 & \text{Log}\left[\text{Sin}\left[\text{ArcTan}\left[\frac{-a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]\right] + \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]\right] + \\
 & i \text{PolyLog}\left[2, e^{2 i \left(\text{ArcTan}\left[\frac{-a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] + \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]}\right)}\right] + \\
 & \frac{1}{4 c (-a c + a^2 c + b^2 d) \sqrt{1 + \frac{(-a c + a^2 c + b^2 d)^2}{b^2 c d}}} b^4 d^2 \left( e^{i \text{ArcTan}\left[\frac{-a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]} \right. \\
 & \left. \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]^2 + \frac{1}{b \sqrt{c} \sqrt{d} \sqrt{1 + \frac{(-a c + a^2 c + b^2 d)^2}{b^2 c d}}} (-a c + a^2 c + b^2 d) \right)
 \end{aligned}$$



$$\left( -\pi \operatorname{Log}\left[1 + e^{-2i \operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]}\right] - i \operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \right) \left( \pi - 2 \operatorname{ArcTan}\left[\frac{-ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] - 2i \operatorname{Log}\left[1 - e^{2i \left(\operatorname{ArcTan}\left[\frac{-ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] + \operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]\right)}\right] \right) -$$

$$2 \operatorname{ArcTan}\left[\frac{-ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \operatorname{Log}\left[1 - e^{2i \left(\operatorname{ArcTan}\left[\frac{-ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] + \operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]\right)}\right] +$$

$$\pi \operatorname{Log}\left[\frac{1}{\sqrt{\frac{(a^2c + b^2d)\left(c + \frac{a^2c + b^2d}{(a+bx)^2} - \frac{2ac}{a+bx}\right)}{b^2cd}}}\right] + 2 \operatorname{ArcTan}\left[\frac{-ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]$$

$$\operatorname{Log}\left[\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{-ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]\right] + \operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]\right] +$$

$$i \operatorname{PolyLog}\left[2, e^{2i \left(\operatorname{ArcTan}\left[\frac{-ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] + \operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]\right)}\right] \Bigg) +$$

$$\frac{1}{2(a^2c + a^2c + b^2d)\sqrt{\frac{b^2cd + (ac + a^2c + b^2d)^2}{b^2cd}}} a^2c \left( e^{-i \operatorname{ArcTan}\left[\frac{ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]} \right)$$

$$\operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]^2 - \frac{1}{b\sqrt{c}\sqrt{d}\sqrt{1 + \frac{(ac + a^2c + b^2d)^2}{b^2cd}}}$$

$$(ac + a^2c + b^2d) \left( i \left( -\pi - 2 \operatorname{ArcTan}\left[\frac{ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \right) \operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] - \right)$$

$$\begin{aligned}
 & \pi \operatorname{Log}\left[1 + e^{-2i \operatorname{ArcTan}\left[\frac{ac - \frac{a^2c + b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]}\right] - 2 \left( -\operatorname{ArcTan}\left[\frac{ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] + \right. \\
 & \left. \operatorname{ArcTan}\left[\frac{ac - \frac{a^2c + b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] \right) \operatorname{Log}\left[1 - e^{2i \left( -\operatorname{ArcTan}\left[\frac{ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] + \operatorname{ArcTan}\left[\frac{ac - \frac{a^2c + b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] \right)}\right] + \\
 & \pi \operatorname{Log}\left[\frac{1}{\sqrt{\frac{(a^2c + b^2d) \left( c + \frac{a^2c + b^2d}{(a+bx)^2} - \frac{2ac}{a+bx} \right)}{b^2cd}}}\right] - 2 \operatorname{ArcTan}\left[\frac{ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \\
 & \operatorname{Log}\left[-\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] - \operatorname{ArcTan}\left[\frac{ac - \frac{a^2c + b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]\right]\right] + \\
 & \left. i \operatorname{PolyLog}\left[2, e^{2i \left( -\operatorname{ArcTan}\left[\frac{ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] + \operatorname{ArcTan}\left[\frac{ac - \frac{a^2c + b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] \right)}\right] \right) + \\
 & \frac{1}{(ac + a^2c + b^2d) \sqrt{\frac{b^2cd + (ac + a^2c + b^2d)^2}{b^2cd}}} a^3c \left( e^{-i \operatorname{ArcTan}\left[\frac{ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]} \right. \\
 & \left. \operatorname{ArcTan}\left[\frac{ac - \frac{a^2c + b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]^2 - \frac{1}{b\sqrt{c}\sqrt{d} \sqrt{1 + \frac{(ac + a^2c + b^2d)^2}{b^2cd}}}\right) \\
 & \left( (ac + a^2c + b^2d) \left( i \left( -\pi - 2 \operatorname{ArcTan}\left[\frac{ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \right) \operatorname{ArcTan}\left[\frac{ac - \frac{a^2c + b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] - \right. \right. \\
 & \left. \left. \pi \operatorname{Log}\left[1 + e^{-2i \operatorname{ArcTan}\left[\frac{ac - \frac{a^2c + b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]}\right] - 2 \left( -\operatorname{ArcTan}\left[\frac{ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] + \right. \right. \right. \\
 & \left. \left. \operatorname{ArcTan}\left[\frac{ac - \frac{a^2c + b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] \right) \operatorname{Log}\left[1 - e^{2i \left( -\operatorname{ArcTan}\left[\frac{ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] + \operatorname{ArcTan}\left[\frac{ac - \frac{a^2c + b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] \right)}\right] + \right.
 \end{aligned}$$

$$\begin{aligned}
 & \pi \operatorname{Log} \left[ \frac{1}{\sqrt{\frac{(a^2 c + b^2 d) \left( c + \frac{a^2 c + b^2 d}{(a+bx)^2} - \frac{2ac}{a-bx} \right)}{b^2 c d}}} \right] - 2 \operatorname{ArcTan} \left[ \frac{ac + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \\
 & \operatorname{Log} \left[ -\operatorname{Sin} \left[ \operatorname{ArcTan} \left[ \frac{ac + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \right] - \operatorname{ArcTan} \left[ \frac{ac - \frac{a^2 c + b^2 d}{a+bx}}{b \sqrt{c} \sqrt{d}} \right] \right] + \\
 & \left. \operatorname{PolyLog} \left[ 2, e^{2i \left( -\operatorname{ArcTan} \left[ \frac{ac + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] + \operatorname{ArcTan} \left[ \frac{ac - \frac{a^2 c + b^2 d}{a+bx}}{b \sqrt{c} \sqrt{d}} \right] \right)} \right] \right) + \\
 & \frac{1}{4 (ac + a^2 c + b^2 d) \sqrt{\frac{b^2 c d + (ac + a^2 c + b^2 d)^2}{b^2 c d}}} \left( 3 a^4 c e^{-i \operatorname{ArcTan} \left[ \frac{ac + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right]} \right. \\
 & \left. \operatorname{ArcTan} \left[ \frac{ac - \frac{a^2 c + b^2 d}{a+bx}}{b \sqrt{c} \sqrt{d}} \right]^2 - \frac{1}{b \sqrt{c} \sqrt{d} \sqrt{1 + \frac{(ac + a^2 c + b^2 d)^2}{b^2 c d}}} \right) \\
 & \left( (ac + a^2 c + b^2 d) \left( i \left( -\pi - 2 \operatorname{ArcTan} \left[ \frac{ac + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \right) \operatorname{ArcTan} \left[ \frac{ac - \frac{a^2 c + b^2 d}{a+bx}}{b \sqrt{c} \sqrt{d}} \right] - \right. \right. \\
 & \left. \left. \pi \operatorname{Log} \left[ 1 + e^{-2i \operatorname{ArcTan} \left[ \frac{ac - \frac{a^2 c + b^2 d}{a+bx}}{b \sqrt{c} \sqrt{d}} \right]} \right] - 2 \left( -\operatorname{ArcTan} \left[ \frac{ac + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] + \right. \right. \right. \\
 & \left. \left. \left. \operatorname{ArcTan} \left[ \frac{ac - \frac{a^2 c + b^2 d}{a+bx}}{b \sqrt{c} \sqrt{d}} \right] \right) \operatorname{Log} \left[ 1 - e^{2i \left( -\operatorname{ArcTan} \left[ \frac{ac + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] + \operatorname{ArcTan} \left[ \frac{ac - \frac{a^2 c + b^2 d}{a+bx}}{b \sqrt{c} \sqrt{d}} \right] \right)} \right] \right) + \\
 & \pi \operatorname{Log} \left[ \frac{1}{\sqrt{\frac{(a^2 c + b^2 d) \left( c + \frac{a^2 c + b^2 d}{(a+bx)^2} - \frac{2ac}{a-bx} \right)}{b^2 c d}}} \right] - 2 \operatorname{ArcTan} \left[ \frac{ac + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right]
 \end{aligned}$$

$$\begin{aligned}
 & \left. \left. \left. \left. \left. \text{Log} \left[ -\text{Sin} \left[ \text{ArcTan} \left[ \frac{a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] - \text{ArcTan} \left[ \frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] \right] \right] \right] \right] \right] + \right. \\
 & \left. \left. \left. \left. \left. i \text{PolyLog} \left[ 2, e^{2 i \left( -\text{ArcTan} \left[ \frac{a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] + \text{ArcTan} \left[ \frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] \right)} \right] \right] \right] \right] \right. + \right. \\
 & \left. \frac{1}{4 b^2 d (a c + a^2 c + b^2 d) \sqrt{\frac{b^2 c d + (a c + a^2 c + b^2 d)^2}{b^2 c d}}} a^4 c^2 \left( e^{-i \text{ArcTan} \left[ \frac{a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right]} \right. \right. \\
 & \left. \left. \text{ArcTan} \left[ \frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right]^2 - \frac{1}{b \sqrt{c} \sqrt{d} \sqrt{1 + \frac{(a c + a^2 c + b^2 d)^2}{b^2 c d}}} \right. \right. \\
 & \left. \left. \left( a c + a^2 c + b^2 d \right) \left( i \left( -\pi - 2 \text{ArcTan} \left[ \frac{a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \right) \text{ArcTan} \left[ \frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] - \right. \right. \\
 & \left. \left. \pi \text{Log} \left[ 1 + e^{-2 i \text{ArcTan} \left[ \frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right]} \right] - 2 \left( -\text{ArcTan} \left[ \frac{a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] + \right. \right. \\
 & \left. \left. \left. \left. \left. \text{ArcTan} \left[ \frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] \right) \text{Log} \left[ 1 - e^{2 i \left( -\text{ArcTan} \left[ \frac{a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] + \text{ArcTan} \left[ \frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] \right)} \right] \right] \right] \right. + \right. \\
 & \left. \left. \pi \text{Log} \left[ \frac{1}{\sqrt{\frac{(a^2 c + b^2 d) \left( c + \frac{a^2 c + b^2 d}{(a + b x)^2} - \frac{2 a c}{a + b x} \right)}{b^2 c d}}} \right] - 2 \text{ArcTan} \left[ \frac{a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \right. \\
 & \left. \left. \text{Log} \left[ -\text{Sin} \left[ \text{ArcTan} \left[ \frac{a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] - \text{ArcTan} \left[ \frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] \right] \right] \right] \right] + \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left. \left. \left. i \operatorname{PolyLog}\left[2, e^{2i \left( -\operatorname{ArcTan}\left[\frac{ac+a^2c+b^2d}{b\sqrt{c}\sqrt{d}}\right] + \operatorname{ArcTan}\left[\frac{ac-\frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]\right)}\right] \right] \right] \right) + \\
 & \frac{1}{2b^2d(ac+a^2c+b^2d)\sqrt{\frac{b^2cd+(ac+a^2c+b^2d)^2}{b^2cd}}} a^5 c^2 \left( e^{-i \operatorname{ArcTan}\left[\frac{ac+a^2c+b^2d}{b\sqrt{c}\sqrt{d}}\right]} \right. \\
 & \left. \operatorname{ArcTan}\left[\frac{ac-\frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]^2 - \frac{1}{b\sqrt{c}\sqrt{d}\sqrt{1+\frac{(ac+a^2c+b^2d)^2}{b^2cd}}} \right. \\
 & \left. (ac+a^2c+b^2d) \left( i \left( -\pi - 2 \operatorname{ArcTan}\left[\frac{ac+a^2c+b^2d}{b\sqrt{c}\sqrt{d}}\right] \right) \operatorname{ArcTan}\left[\frac{ac-\frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] - \right. \right. \\
 & \left. \left. \pi \operatorname{Log}\left[1 + e^{-2i \operatorname{ArcTan}\left[\frac{ac-\frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]}\right] - 2 \left( -\operatorname{ArcTan}\left[\frac{ac+a^2c+b^2d}{b\sqrt{c}\sqrt{d}}\right] + \right. \right. \right. \\
 & \left. \left. \left. \operatorname{ArcTan}\left[\frac{ac-\frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] \right) \operatorname{Log}\left[1 - e^{2i \left( -\operatorname{ArcTan}\left[\frac{ac+a^2c+b^2d}{b\sqrt{c}\sqrt{d}}\right] + \operatorname{ArcTan}\left[\frac{ac-\frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]\right)}\right] \right) \right] + \\
 & \left. \left. \left. \pi \operatorname{Log}\left[\frac{1}{\sqrt{\frac{(a^2c+b^2d)\left(c+\frac{a^2c+b^2d}{(a+bx)^2}-\frac{2ac}{a+bx}\right)}{b^2cd}}}\right] - 2 \operatorname{ArcTan}\left[\frac{ac+a^2c+b^2d}{b\sqrt{c}\sqrt{d}}\right] \right. \right. \right. \\
 & \left. \left. \left. \operatorname{Log}\left[-\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{ac+a^2c+b^2d}{b\sqrt{c}\sqrt{d}}\right]\right] - \operatorname{ArcTan}\left[\frac{ac-\frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]\right] \right] \right] \right) + \\
 & \left. \left. \left. i \operatorname{PolyLog}\left[2, e^{2i \left( -\operatorname{ArcTan}\left[\frac{ac+a^2c+b^2d}{b\sqrt{c}\sqrt{d}}\right] + \operatorname{ArcTan}\left[\frac{ac-\frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]\right)}\right] \right] \right] \right) +
 \end{aligned}$$

$$\begin{aligned}
 & \frac{1}{4 b^2 d (a c + a^2 c + b^2 d) \sqrt{\frac{b^2 c d + (a c + a^2 c + b^2 d)^2}{b^2 c d}}} a^6 c^2 \left( e^{-i \operatorname{ArcTan}\left[\frac{a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]} \right. \\
 & \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]^2 - \frac{1}{b \sqrt{c} \sqrt{d} \sqrt{1 + \frac{(a c + a^2 c + b^2 d)^2}{b^2 c d}}} \\
 & (a c + a^2 c + b^2 d) \left( i \left( -\pi - 2 \operatorname{ArcTan}\left[\frac{a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] \right) \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] - \right. \\
 & \left. \pi \operatorname{Log}\left[1 + e^{-2 i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]}\right] - 2 \left( -\operatorname{ArcTan}\left[\frac{a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] + \right. \right. \\
 & \left. \left. \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] \right) \operatorname{Log}\left[1 - e^{2 i \left( -\operatorname{ArcTan}\left[\frac{a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] + \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] \right)}\right] + \right. \\
 & \left. \pi \operatorname{Log}\left[\frac{1}{\sqrt{\frac{(a^2 c + b^2 d) \left( c + \frac{a^2 c + b^2 d}{(a + b x)^2} - \frac{2 a c}{a + b x} \right)}{b^2 c d}}}\right] - 2 \operatorname{ArcTan}\left[\frac{a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] \right. \\
 & \left. \operatorname{Log}\left[-\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] - \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]\right]\right] + \right. \\
 & \left. i \operatorname{PolyLog}\left[2, e^{2 i \left( -\operatorname{ArcTan}\left[\frac{a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] + \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] \right)}\right] \right) + \\
 & \frac{1}{4 (a c + a^2 c + b^2 d) \sqrt{\frac{b^2 c d + (a c + a^2 c + b^2 d)^2}{b^2 c d}}} b^2 d \left( e^{-i \operatorname{ArcTan}\left[\frac{a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]} \right.
 \end{aligned}$$

$$\begin{aligned}
 & \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]^2 - \frac{1}{b \sqrt{c} \sqrt{d} \sqrt{1 + \frac{(a c + a^2 c + b^2 d)^2}{b^2 c d}}} \\
 & (a c + a^2 c + b^2 d) \left( i \left( -\pi - 2 \text{ArcTan}\left[\frac{a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] \right) \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] - \right. \\
 & \left. \pi \text{Log}\left[1 + e^{-2 i \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]}\right] - 2 \left( -\text{ArcTan}\left[\frac{a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] + \right. \right. \\
 & \left. \left. \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] \right) \text{Log}\left[1 - e^{2 i \left( -\text{ArcTan}\left[\frac{a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] + \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] \right)}\right] + \right. \\
 & \left. \pi \text{Log}\left[\frac{1}{\sqrt{\frac{(a^2 c + b^2 d) \left( c + \frac{a^2 c + b^2 d}{(a + b x)^2} \frac{2 a c}{a + b x} \right)}{b^2 c d}}}\right] - 2 \text{ArcTan}\left[\frac{a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] \right. \\
 & \left. \text{Log}\left[-\text{Sin}\left[\text{ArcTan}\left[\frac{a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] - \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]\right]\right] + \right. \\
 & \left. i \text{PolyLog}\left[2, e^{2 i \left( -\text{ArcTan}\left[\frac{a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] + \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] \right)}\right] \right) \left. \right) + \\
 & \frac{1}{2 (a c + a^2 c + b^2 d) \sqrt{\frac{b^2 c d + (a c + a^2 c + b^2 d)^2}{b^2 c d}}} a b^2 d \left( e^{-i \text{ArcTan}\left[\frac{a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]} \right. \\
 & \left. \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]^2 - \frac{1}{b \sqrt{c} \sqrt{d} \sqrt{1 + \frac{(a c + a^2 c + b^2 d)^2}{b^2 c d}}} \right)
 \end{aligned}$$

$$\begin{aligned}
 & (a c + a^2 c + b^2 d) \left( i \left( -\pi - 2 \operatorname{ArcTan} \left[ \frac{a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \right) \operatorname{ArcTan} \left[ \frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] - \right. \\
 & \pi \operatorname{Log} \left[ 1 + e^{-2 i \operatorname{ArcTan} \left[ \frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right]} \right] - 2 \left( -\operatorname{ArcTan} \left[ \frac{a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] + \right. \\
 & \left. \left. \operatorname{ArcTan} \left[ \frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] \right) \operatorname{Log} \left[ 1 - e^{2 i \left( -\operatorname{ArcTan} \left[ \frac{a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] + \operatorname{ArcTan} \left[ \frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] \right)} \right] + \\
 & \pi \operatorname{Log} \left[ \frac{1}{\sqrt{\frac{(a^2 c + b^2 d) \left( c + \frac{a^2 c + b^2 d}{(a + b x)^2} \frac{2 a c}{a + b x} \right)}{b^2 c d}}} \right] - 2 \operatorname{ArcTan} \left[ \frac{a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \\
 & \operatorname{Log} \left[ -\operatorname{Sin} \left[ \operatorname{ArcTan} \left[ \frac{a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] - \operatorname{ArcTan} \left[ \frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] \right] \right] + \\
 & \left. \left. \left. i \operatorname{PolyLog} \left[ 2, e^{2 i \left( -\operatorname{ArcTan} \left[ \frac{a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] + \operatorname{ArcTan} \left[ \frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] \right)} \right] \right] \right) \right) + \\
 & \frac{1}{4 (a c + a^2 c + b^2 d) \sqrt{\frac{b^2 c d + (a c + a^2 c + b^2 d)^2}{b^2 c d}}} 3 a^2 b^2 d \left( e^{-i \operatorname{ArcTan} \left[ \frac{a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right]} \right. \\
 & \left. \operatorname{ArcTan} \left[ \frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right]^2 - \frac{1}{b \sqrt{c} \sqrt{d} \sqrt{1 + \frac{(a c + a^2 c + b^2 d)^2}{b^2 c d}}} \right) \\
 & (a c + a^2 c + b^2 d) \left( i \left( -\pi - 2 \operatorname{ArcTan} \left[ \frac{a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \right) \operatorname{ArcTan} \left[ \frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] - \right.
 \end{aligned}$$



$$\begin{aligned}
 & \pi \operatorname{Log}\left[1 + e^{-2i \operatorname{ArcTan}\left[\frac{ac - \frac{a^2c + b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]}\right] - 2 \left( -\operatorname{ArcTan}\left[\frac{ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] + \right. \\
 & \quad \left. \operatorname{ArcTan}\left[\frac{ac - \frac{a^2c + b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] \right) \operatorname{Log}\left[1 - e^{2i \left( -\operatorname{ArcTan}\left[\frac{ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] + \operatorname{ArcTan}\left[\frac{ac - \frac{a^2c + b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] \right)}\right] + \\
 & \pi \operatorname{Log}\left[\frac{1}{\sqrt{\frac{(a^2c + b^2d) \left( c + \frac{a^2c + b^2d}{(a+bx)^2} - \frac{2ac}{a+bx} \right)}{b^2cd}}}\right] - 2 \operatorname{ArcTan}\left[\frac{ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \\
 & \operatorname{Log}\left[-\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] - \operatorname{ArcTan}\left[\frac{ac - \frac{a^2c + b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]\right]\right] + \\
 & i \operatorname{PolyLog}\left[2, e^{2i \left( -\operatorname{ArcTan}\left[\frac{ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] + \operatorname{ArcTan}\left[\frac{ac - \frac{a^2c + b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] \right)}\right] \left. \right) + \\
 & \frac{1}{4c(ac + a^2c + b^2d)\sqrt{\frac{b^2cd + (ac + a^2c + b^2d)^2}{b^2cd}}} b^4 d^2 \left( e^{-i \operatorname{ArcTan}\left[\frac{ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]} \right. \\
 & \operatorname{ArcTan}\left[\frac{ac - \frac{a^2c + b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]^2 - \frac{1}{b\sqrt{c}\sqrt{d}\sqrt{1 + \frac{(ac + a^2c + b^2d)^2}{b^2cd}}} \\
 & \left. (ac + a^2c + b^2d) \left( i \left( -\pi - 2 \operatorname{ArcTan}\left[\frac{ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \right) \operatorname{ArcTan}\left[\frac{ac - \frac{a^2c + b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] - \right. \right. \\
 & \left. \left. \pi \operatorname{Log}\left[1 + e^{-2i \operatorname{ArcTan}\left[\frac{ac - \frac{a^2c + b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]}\right] - 2 \left( -\operatorname{ArcTan}\left[\frac{ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] + \right. \right. \right. \\
 & \left. \left. \left. \operatorname{ArcTan}\left[\frac{ac - \frac{a^2c + b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] \right) \operatorname{Log}\left[1 - e^{2i \left( -\operatorname{ArcTan}\left[\frac{ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] + \operatorname{ArcTan}\left[\frac{ac - \frac{a^2c + b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] \right)}\right] \right) + \right.
 \end{aligned}$$

$$\pi \operatorname{Log}\left[\frac{1}{\sqrt{\frac{(a^2 c + b^2 d) \left(c + \frac{a^2 c + b^2 d}{(a + b x)^2} - \frac{2 a c}{a - b x}\right)}{b^2 c d}}}\right] - 2 \operatorname{ArcTan}\left[\frac{a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]$$

$$\operatorname{Log}\left[-\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]\right] - \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]\right] +$$

$$i \operatorname{PolyLog}\left[2, e^{2 i \left(-\operatorname{ArcTan}\left[\frac{a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] + \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]\right)}\right]$$

**Problem 80: Unable to integrate problem.**

$$\int \frac{\operatorname{ArcCoth}[a + b x]}{c + d \sqrt{x}} dx$$

Optimal (type 4, 619 leaves, 55 steps):

$$\frac{2 \sqrt{1+a} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{x}}{\sqrt{1+a}}\right] - 2 \sqrt{1-a} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{x}}{\sqrt{1-a}}\right] + \frac{c \operatorname{Log}\left[\frac{d(\sqrt{-1-a}-\sqrt{b} \sqrt{x})}{\sqrt{b} c + \sqrt{-1-a} d}\right] \operatorname{Log}[c + d \sqrt{x}]}{d^2}}{\sqrt{b} d} - \frac{c \operatorname{Log}\left[\frac{d(\sqrt{-1-a}-\sqrt{b} \sqrt{x})}{\sqrt{b} c + \sqrt{-1-a} d}\right] \operatorname{Log}[c + d \sqrt{x}]}{d^2} + \frac{c \operatorname{Log}\left[-\frac{d(\sqrt{-1-a} + \sqrt{b} \sqrt{x})}{\sqrt{b} c - \sqrt{-1-a} d}\right] \operatorname{Log}[c + d \sqrt{x}]}{d^2} - \frac{c \operatorname{Log}\left[-\frac{d(\sqrt{-1-a} + \sqrt{b} \sqrt{x})}{\sqrt{b} c - \sqrt{-1-a} d}\right] \operatorname{Log}[c + d \sqrt{x}]}{d^2} - \frac{\sqrt{x} \operatorname{Log}\left[-\frac{1-a-bx}{a+bx}\right]}{d} + \frac{c \operatorname{Log}[c + d \sqrt{x}] \operatorname{Log}\left[-\frac{1-a-bx}{a+bx}\right]}{d^2} + \frac{\sqrt{x} \operatorname{Log}\left[\frac{1+a+bx}{a+bx}\right]}{d} - \frac{c \operatorname{Log}[c + d \sqrt{x}] \operatorname{Log}\left[\frac{1+a+bx}{a+bx}\right]}{d^2} + \frac{c \operatorname{PolyLog}\left[2, \frac{\sqrt{b}(c+d \sqrt{x})}{\sqrt{b} c - \sqrt{-1-a} d}\right]}{d^2} + \frac{c \operatorname{PolyLog}\left[2, \frac{\sqrt{b}(c+d \sqrt{x})}{\sqrt{b} c + \sqrt{-1-a} d}\right]}{d^2} - \frac{c \operatorname{PolyLog}\left[2, \frac{\sqrt{b}(c+d \sqrt{x})}{\sqrt{b} c - \sqrt{-1-a} d}\right]}{d^2} - \frac{c \operatorname{PolyLog}\left[2, \frac{\sqrt{b}(c+d \sqrt{x})}{\sqrt{b} c + \sqrt{-1-a} d}\right]}{d^2}$$

Result (type 8, 20 leaves):

$$\int \frac{\operatorname{ArcCoth}[a + b x]}{c + d \sqrt{x}} dx$$

Problem 81: Attempted integration timed out after 120 seconds.

$$\int \frac{\text{ArcCoth}[a + b x]}{c + \frac{d}{\sqrt{x}}} dx$$

Optimal (type 4, 738 leaves, 65 steps):

$$\begin{aligned} & -\frac{2\sqrt{1+a} d \text{ArcTan}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right]}{\sqrt{b} c^2} + \frac{2\sqrt{1-a} d \text{ArcTanh}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right]}{\sqrt{b} c^2} \\ & -\frac{d^2 \text{Log}\left[\frac{c(\sqrt{-1-a}-\sqrt{b}\sqrt{x})}{\sqrt{-1-a}c+\sqrt{b}d}\right] \text{Log}[d+c\sqrt{x}]}{c^3} + \frac{d^2 \text{Log}\left[\frac{c(\sqrt{1-a}-\sqrt{b}\sqrt{x})}{\sqrt{1-a}c+\sqrt{b}d}\right] \text{Log}[d+c\sqrt{x}]}{c^3} \\ & -\frac{d^2 \text{Log}\left[\frac{c(\sqrt{-1-a}+\sqrt{b}\sqrt{x})}{\sqrt{-1-a}c-\sqrt{b}d}\right] \text{Log}[d+c\sqrt{x}]}{c^3} + \frac{d^2 \text{Log}\left[\frac{c(\sqrt{1-a}+\sqrt{b}\sqrt{x})}{\sqrt{1-a}c-\sqrt{b}d}\right] \text{Log}[d+c\sqrt{x}]}{c^3} \\ & -\frac{(1-a) \text{Log}[1-a-bx]}{2bc} + \frac{d\sqrt{x} \text{Log}\left[-\frac{1-a-bx}{a+bx}\right]}{c^2} - \frac{x \text{Log}\left[-\frac{1-a-bx}{a+bx}\right]}{2c} \\ & -\frac{d^2 \text{Log}[d+c\sqrt{x}] \text{Log}\left[-\frac{1-a-bx}{a+bx}\right]}{c^3} + \frac{(1+a) \text{Log}[1+a+bx]}{2bc} - \frac{d\sqrt{x} \text{Log}\left[\frac{1+a+bx}{a+bx}\right]}{c^2} \\ & + \frac{x \text{Log}\left[\frac{1+a+bx}{a+bx}\right]}{2c} + \frac{d^2 \text{Log}[d+c\sqrt{x}] \text{Log}\left[\frac{1+a+bx}{a+bx}\right]}{c^3} - \frac{d^2 \text{PolyLog}\left[2, -\frac{\sqrt{b}(d+c\sqrt{x})}{\sqrt{-1-a}c-\sqrt{b}d}\right]}{c^3} \\ & -\frac{d^2 \text{PolyLog}\left[2, -\frac{\sqrt{b}(d+c\sqrt{x})}{\sqrt{1-a}c+\sqrt{b}d}\right]}{c^3} - \frac{d^2 \text{PolyLog}\left[2, \frac{\sqrt{b}(d+c\sqrt{x})}{\sqrt{-1-a}c+\sqrt{b}d}\right]}{c^3} + \frac{d^2 \text{PolyLog}\left[2, \frac{\sqrt{b}(d+c\sqrt{x})}{\sqrt{1-a}c+\sqrt{b}d}\right]}{c^3} \end{aligned}$$

Result (type 1, 1 leaves):

???

Problem 82: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcCoth}[d + e x]}{a + b x + c x^2} dx$$

Optimal (type 4, 335 leaves, 12 steps):

$$\frac{\text{ArcCoth}[d + e x] \text{Log}\left[\frac{2 e \left(b - \sqrt{b^2 - 4 a c} + 2 c x\right)}{\left(2 c (1 - d) + \left(b - \sqrt{b^2 - 4 a c}\right) e\right) (1 + d + e x)}\right]}{\sqrt{b^2 - 4 a c}} -$$

$$\frac{\text{ArcCoth}[d + e x] \text{Log}\left[\frac{2 e \left(b + \sqrt{b^2 - 4 a c} + 2 c x\right)}{\left(2 c (1 - d) + \left(b + \sqrt{b^2 - 4 a c}\right) e\right) (1 + d + e x)}\right]}{\sqrt{b^2 - 4 a c}} -$$

$$\frac{\text{PolyLog}\left[2, 1 + \frac{2 \left(2 c d - \left(b - \sqrt{b^2 - 4 a c}\right) e - 2 c (d + e x)\right)}{\left(2 c - 2 c d + b e - \sqrt{b^2 - 4 a c} e\right) (1 + d + e x)}\right]}{2 \sqrt{b^2 - 4 a c}} + \frac{\text{PolyLog}\left[2, 1 + \frac{2 \left(2 c d - \left(b + \sqrt{b^2 - 4 a c}\right) e - 2 c (d + e x)\right)}{\left(2 c (1 - d) + \left(b + \sqrt{b^2 - 4 a c}\right) e\right) (1 + d + e x)}\right]}{2 \sqrt{b^2 - 4 a c}}$$

Result (type 4, 8833 leaves):

$$-\frac{1}{e (d + e x)^2 (a + b x + c x^2) \left(1 - \frac{1}{(d + e x)^2}\right)} (a e + b e x + c e x^2)$$

$$\left(1 - (d + e x)^2\right) \left[ -\frac{2 \text{ArcCoth}[d + e x] \text{ArcTanh}\left[\frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c} e}\right]}{\sqrt{b^2 - 4 a c}} - \frac{1}{c (-1 + (d + e x)^2)} \right]$$

$$e \left( -1 + \frac{1}{4 c^2} \left( 2 c d - b e + \sqrt{b^2 - 4 a c} e \left( \frac{b}{\sqrt{b^2 - 4 a c}} - \frac{2 c d}{\sqrt{b^2 - 4 a c} e} + \frac{2 c (d + e x)}{\sqrt{b^2 - 4 a c} e} \right) \right) \right)^2$$

$$\left[ \frac{2 c^2 \text{ArcTanh}\left[\frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c} e}\right]^2}{4 c^2 (-1 + d^2) - 4 b c d e + b^2 e^2} + \right.$$

$$\left. \frac{1}{(b^2 - 4 a c) (2 c - 2 c d + b e) \sqrt{\frac{(b^2 - 4 a c) e^2 - (2 c (-1 + d) - b e)^2}{(b^2 - 4 a c) e^2}}} 2 a c^2 \left[ -e^{-\text{ArcTanh}\left[\frac{2 c (-1 + d) - b e}{\sqrt{b^2 - 4 a c} e}\right]} \right. \right.$$

$$\left. \text{ArcTanh}\left[\frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c} e}\right]^2 + \frac{1}{\sqrt{b^2 - 4 a c} e \sqrt{1 - \frac{(2 c (-1 + d) - b e)^2}{(b^2 - 4 a c) e^2}}} \right]$$

$$\begin{aligned}
 & i (2c(-1+d) - be) \left( - \left( -\pi + 2i \operatorname{ArcTanh} \left[ \frac{2c(-1+d) - be}{\sqrt{b^2 - 4ace}} \right] \right) \right. \\
 & \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ace}} \right] - \pi \operatorname{Log} \left[ 1 + e^{\frac{2 \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ace}} \right]}{\sqrt{b^2 - 4ace}}} \right] - \\
 & 2 \left( i \operatorname{ArcTanh} \left[ \frac{2c(-1+d) - be}{\sqrt{b^2 - 4ace}} \right] + i \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ace}} \right] \right) \\
 & \operatorname{Log} \left[ 1 - e^{-2 \left( \operatorname{ArcTanh} \left[ \frac{2c(-1+d) - be}{\sqrt{b^2 - 4ace}} \right] + \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ace}} \right] \right)} \right] + \\
 & \pi \operatorname{Log} \left[ \frac{1}{\sqrt{1 - \left( \frac{b}{\sqrt{b^2 - 4ac}} - \frac{2cd}{\sqrt{b^2 - 4ace}} + \frac{2c(d+ex)}{\sqrt{b^2 - 4ace}} \right)^2}} \right] + \\
 & 2i \operatorname{ArcTanh} \left[ \frac{2c(-1+d) - be}{\sqrt{b^2 - 4ace}} \right] \operatorname{Log} \left[ \right. \\
 & \left. i \operatorname{Sinh} \left[ \operatorname{ArcTanh} \left[ \frac{2c(-1+d) - be}{\sqrt{b^2 - 4ace}} \right] + \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ace}} \right] \right] \right] + \\
 & \left. i \operatorname{PolyLog} \left[ 2, e^{-2 \left( \operatorname{ArcTanh} \left[ \frac{2c(-1+d) - be}{\sqrt{b^2 - 4ace}} \right] + \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ace}} \right] \right)} \right] \right) \right) + \\
 & \frac{1}{(b^2 - 4ac) e^2 (2c - 2cd + be) \sqrt{\frac{(b^2 - 4ac) e^2 - (2c(-1+d) - be)^2}{(b^2 - 4ac) e^2}}} 2c^3 \left( -e^{-\operatorname{ArcTanh} \left[ \frac{2c(-1+d) - be}{\sqrt{b^2 - 4ace}} \right]} \right. \\
 & \left. \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ace}} \right] \right)^2 + \frac{1}{\sqrt{b^2 - 4ace} e \sqrt{1 - \frac{(2c(-1+d) - be)^2}{(b^2 - 4ac) e^2}}}
 \end{aligned}$$

$$\begin{aligned}
 & i (2c(-1+d) - be) \left( - \left( -\pi + 2i \operatorname{ArcTanh} \left[ \frac{2c(-1+d) - be}{\sqrt{b^2 - 4ace}} \right] \right) \right. \\
 & \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ace}} \right] - \pi \operatorname{Log} \left[ 1 + e^{\frac{2 \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ace}} \right]}{\sqrt{b^2 - 4ace}}} \right] - \\
 & 2 \left( i \operatorname{ArcTanh} \left[ \frac{2c(-1+d) - be}{\sqrt{b^2 - 4ace}} \right] + i \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ace}} \right] \right) \\
 & \operatorname{Log} \left[ 1 - e^{-2 \left( \operatorname{ArcTanh} \left[ \frac{2c(-1+d) - be}{\sqrt{b^2 - 4ace}} \right] + \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ace}} \right] \right)} \right] + \\
 & \pi \operatorname{Log} \left[ \frac{1}{\sqrt{1 - \left( \frac{b}{\sqrt{b^2 - 4ac}} - \frac{2cd}{\sqrt{b^2 - 4ace}} + \frac{2c(d+ex)}{\sqrt{b^2 - 4ace}} \right)^2}} \right] + \\
 & 2i \operatorname{ArcTanh} \left[ \frac{2c(-1+d) - be}{\sqrt{b^2 - 4ace}} \right] \operatorname{Log} \left[ \right. \\
 & \left. i \operatorname{Sinh} \left[ \operatorname{ArcTanh} \left[ \frac{2c(-1+d) - be}{\sqrt{b^2 - 4ace}} \right] + \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ace}} \right] \right] \right] + \\
 & \left. i \operatorname{PolyLog} \left[ 2, e^{-2 \left( \operatorname{ArcTanh} \left[ \frac{2c(-1+d) - be}{\sqrt{b^2 - 4ace}} \right] + \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ace}} \right] \right)} \right] \right) \right) \\
 & \frac{1}{(b^2 - 4ac) e^2 (2c - 2cd + be) \sqrt{\frac{(b^2 - 4ac) e^2 - (2c(-1+d) - be)^2}{(b^2 - 4ac) e^2}}} \\
 & 4c^3 d \left( -e^{-\operatorname{ArcTanh} \left[ \frac{2c(-1+d) - be}{\sqrt{b^2 - 4ace}} \right]} \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ace}} \right]^2 + \right. \\
 & \left. \frac{1}{\sqrt{b^2 - 4ac} e} \sqrt{1 - \frac{(2c(-1+d) - be)^2}{(b^2 - 4ac) e^2}} \right)
 \end{aligned}$$

$$\begin{aligned}
 & i (2c(-1+d) - be) \left( - \left( -\pi + 2i \operatorname{ArcTanh} \left[ \frac{2c(-1+d) - be}{\sqrt{b^2 - 4ac}e} \right] \right) \right. \\
 & \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac}e} \right] - \pi \operatorname{Log} \left[ 1 + e^{\frac{2 \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac}e} \right]}{}} \right] - \\
 & 2 \left( i \operatorname{ArcTanh} \left[ \frac{2c(-1+d) - be}{\sqrt{b^2 - 4ac}e} \right] + i \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac}e} \right] \right) \\
 & \operatorname{Log} \left[ 1 - e^{-2 \left( \operatorname{ArcTanh} \left[ \frac{2c(-1+d) - be}{\sqrt{b^2 - 4ac}e} \right] + \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac}e} \right] \right)} \right] + \\
 & \pi \operatorname{Log} \left[ \frac{1}{\sqrt{1 - \left( \frac{b}{\sqrt{b^2 - 4ac}} - \frac{2cd}{\sqrt{b^2 - 4ac}e} + \frac{2c(d+ex)}{\sqrt{b^2 - 4ac}e} \right)^2}} \right] + \\
 & 2i \operatorname{ArcTanh} \left[ \frac{2c(-1+d) - be}{\sqrt{b^2 - 4ac}e} \right] \operatorname{Log} \left[ \right. \\
 & \left. i \operatorname{Sinh} \left[ \operatorname{ArcTanh} \left[ \frac{2c(-1+d) - be}{\sqrt{b^2 - 4ac}e} \right] + \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac}e} \right] \right] \right] + \\
 & \left. i \operatorname{PolyLog} \left[ 2, e^{-2 \left( \operatorname{ArcTanh} \left[ \frac{2c(-1+d) - be}{\sqrt{b^2 - 4ac}e} \right] + \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac}e} \right] \right)} \right] \right] \right) +
 \end{aligned}$$

$$\frac{1}{(b^2 - 4ac) e^2 (2c - 2cd + be) \sqrt{\frac{(b^2 - 4ac) e^2 - (2c(-1+d) - be)^2}{(b^2 - 4ac) e^2}}}$$

$$2c^3 d^2 \left( -e^{-\operatorname{ArcTanh} \left[ \frac{2c(-1+d) - be}{\sqrt{b^2 - 4ac}e} \right]} \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac}e} \right] \right)^2 +$$

$$\frac{1}{\sqrt{b^2 - 4ac} e \sqrt{1 - \frac{(2c(-1+d) - be)^2}{(b^2 - 4ac) e^2}}}$$

$$\begin{aligned}
 & i (2 c (-1 + d) - b e) \left( - \left( -\pi + 2 i \operatorname{ArcTanh} \left[ \frac{2 c (-1 + d) - b e}{\sqrt{b^2 - 4 a c e}} \right] \right) \right. \\
 & \operatorname{ArcTanh} \left[ \frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c e}} \right] - \pi \operatorname{Log} \left[ 1 + e^{\frac{2 \operatorname{ArcTanh} \left[ \frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c e}} \right]}{\sqrt{b^2 - 4 a c e}}} \right] - \\
 & 2 \left( i \operatorname{ArcTanh} \left[ \frac{2 c (-1 + d) - b e}{\sqrt{b^2 - 4 a c e}} \right] + i \operatorname{ArcTanh} \left[ \frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c e}} \right] \right) \\
 & \operatorname{Log} \left[ 1 - e^{-2 \left( \operatorname{ArcTanh} \left[ \frac{2 c (-1 + d) - b e}{\sqrt{b^2 - 4 a c e}} \right] + \operatorname{ArcTanh} \left[ \frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c e}} \right] \right)} \right] + \\
 & \pi \operatorname{Log} \left[ \frac{1}{\sqrt{1 - \left( \frac{b}{\sqrt{b^2 - 4 a c}} - \frac{2 c d}{\sqrt{b^2 - 4 a c e}} + \frac{2 c (d + e x)}{\sqrt{b^2 - 4 a c e}} \right)^2}} \right] + \\
 & 2 i \operatorname{ArcTanh} \left[ \frac{2 c (-1 + d) - b e}{\sqrt{b^2 - 4 a c e}} \right] \operatorname{Log} \left[ \right. \\
 & \left. i \operatorname{Sinh} \left[ \operatorname{ArcTanh} \left[ \frac{2 c (-1 + d) - b e}{\sqrt{b^2 - 4 a c e}} \right] + \operatorname{ArcTanh} \left[ \frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c e}} \right] \right] \right] + \\
 & \left. i \operatorname{PolyLog} \left[ 2, e^{-2 \left( \operatorname{ArcTanh} \left[ \frac{2 c (-1 + d) - b e}{\sqrt{b^2 - 4 a c e}} \right] + \operatorname{ArcTanh} \left[ \frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c e}} \right] \right)} \right] \right) \right) + \\
 & \frac{1}{(b^2 - 4 a c) e (2 c - 2 c d + b e) \sqrt{\frac{(b^2 - 4 a c) e^2 - (2 c (-1 + d) - b e)^2}{(b^2 - 4 a c) e^2}}} \\
 & 2 b c^2 \left( - e^{-\operatorname{ArcTanh} \left[ \frac{2 c (-1 + d) - b e}{\sqrt{b^2 - 4 a c e}} \right]} \operatorname{ArcTanh} \left[ \frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c e}} \right] \right)^2 + \\
 & \frac{1}{\sqrt{b^2 - 4 a c} e \sqrt{1 - \frac{(2 c (-1 + d) - b e)^2}{(b^2 - 4 a c) e^2}}}
 \end{aligned}$$



$$\begin{aligned}
 & i (2 c (-1 + d) - b e) \left( - \left( -\pi + 2 i \operatorname{ArcTanh} \left[ \frac{2 c (-1 + d) - b e}{\sqrt{b^2 - 4 a c e}} \right] \right) \right. \\
 & \operatorname{ArcTanh} \left[ \frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c e}} \right] - \pi \operatorname{Log} \left[ 1 + e^{\frac{2 \operatorname{ArcTanh} \left[ \frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c e}} \right]}{\sqrt{b^2 - 4 a c e}}} \right] - \\
 & 2 \left( i \operatorname{ArcTanh} \left[ \frac{2 c (-1 + d) - b e}{\sqrt{b^2 - 4 a c e}} \right] + i \operatorname{ArcTanh} \left[ \frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c e}} \right] \right) \\
 & \operatorname{Log} \left[ 1 - e^{-2 \left( \operatorname{ArcTanh} \left[ \frac{2 c (-1 + d) - b e}{\sqrt{b^2 - 4 a c e}} \right] + \operatorname{ArcTanh} \left[ \frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c e}} \right] \right)} \right] + \\
 & \pi \operatorname{Log} \left[ \frac{1}{\sqrt{1 - \left( \frac{b}{\sqrt{b^2 - 4 a c}} - \frac{2 c d}{\sqrt{b^2 - 4 a c e}} + \frac{2 c (d + e x)}{\sqrt{b^2 - 4 a c e}} \right)^2}} \right] + \\
 & 2 i \operatorname{ArcTanh} \left[ \frac{2 c (-1 + d) - b e}{\sqrt{b^2 - 4 a c e}} \right] \operatorname{Log} \left[ \right. \\
 & \left. i \operatorname{Sinh} \left[ \operatorname{ArcTanh} \left[ \frac{2 c (-1 + d) - b e}{\sqrt{b^2 - 4 a c e}} \right] + \operatorname{ArcTanh} \left[ \frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c e}} \right] \right] \right] + \\
 & i \operatorname{PolyLog} \left[ 2, e^{-2 \left( \operatorname{ArcTanh} \left[ \frac{2 c (-1 + d) - b e}{\sqrt{b^2 - 4 a c e}} \right] + \operatorname{ArcTanh} \left[ \frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c e}} \right] \right)} \right] \left. \right) -
 \end{aligned}$$

$$\frac{1}{(b^2 - 4 a c) e (2 c - 2 c d + b e)} \sqrt{\frac{(b^2 - 4 a c) e^2 - (2 c (-1 + d) - b e)^2}{(b^2 - 4 a c) e^2}} 2 b c^2 d$$

$$\left( -e^{-\operatorname{ArcTanh} \left[ \frac{2 c (-1 + d) - b e}{\sqrt{b^2 - 4 a c e}} \right]} \operatorname{ArcTanh} \left[ \frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c e}} \right] \right)^2 +$$

$$\frac{1}{\sqrt{b^2 - 4 a c} e} \sqrt{1 - \frac{(2 c (-1 + d) - b e)^2}{(b^2 - 4 a c) e^2}}$$

$$\begin{aligned}
 & i (2c(-1+d) - be) \left( - \left( -\pi + 2i \operatorname{ArcTanh} \left[ \frac{2c(-1+d) - be}{\sqrt{b^2 - 4ac}e} \right] \right) \right. \\
 & \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac}e} \right] - \pi \operatorname{Log} \left[ 1 + e^{\frac{2 \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac}e} \right]}{\sqrt{b^2 - 4ac}e}} \right] - \\
 & 2 \left( i \operatorname{ArcTanh} \left[ \frac{2c(-1+d) - be}{\sqrt{b^2 - 4ac}e} \right] + i \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac}e} \right] \right) \\
 & \operatorname{Log} \left[ 1 - e^{-2 \left( \operatorname{ArcTanh} \left[ \frac{2c(-1+d) - be}{\sqrt{b^2 - 4ac}e} \right] + \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac}e} \right] \right)} \right] + \\
 & \pi \operatorname{Log} \left[ \frac{1}{\sqrt{1 - \left( \frac{b}{\sqrt{b^2 - 4ac}} - \frac{2cd}{\sqrt{b^2 - 4ac}e} + \frac{2c(d+ex)}{\sqrt{b^2 - 4ac}e} \right)^2}} \right] + \\
 & 2i \operatorname{ArcTanh} \left[ \frac{2c(-1+d) - be}{\sqrt{b^2 - 4ac}e} \right] \operatorname{Log} \left[ \right. \\
 & \left. i \operatorname{Sinh} \left[ \operatorname{ArcTanh} \left[ \frac{2c(-1+d) - be}{\sqrt{b^2 - 4ac}e} \right] + \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac}e} \right] \right] \right] + \\
 & \left. i \operatorname{PolyLog} \left[ 2, e^{-2 \left( \operatorname{ArcTanh} \left[ \frac{2c(-1+d) - be}{\sqrt{b^2 - 4ac}e} \right] + \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac}e} \right] \right)} \right] \right) \right) \\
 & \frac{1}{(b^2 - 4ac)(-2c - 2cd + be) \sqrt{\frac{(b^2 - 4ac)e^2 - (2c(1+d) - be)^2}{(b^2 - 4ac)e^2}}} \\
 & 2ac^2 \left( -e^{-\operatorname{ArcTanh} \left[ \frac{2c(1+d) - be}{\sqrt{b^2 - 4ac}e} \right]} \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac}e} \right] \right)^2 + \\
 & \frac{1}{\sqrt{b^2 - 4ac}e \sqrt{1 - \frac{(2c(1+d) - be)^2}{(b^2 - 4ac)e^2}}}
 \end{aligned}$$

$$\begin{aligned}
 & i (2c(1+d) - be) \left( -\left( -\pi + 2i \operatorname{ArcTanh} \left[ \frac{2c(1+d) - be}{\sqrt{b^2 - 4ace}} \right] \right) \right. \\
 & \quad \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ace}} \right] - \pi \operatorname{Log} \left[ 1 + e^{\frac{2 \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ace}} \right]}{\sqrt{b^2 - 4ace}}} \right] - \\
 & \quad 2 \left( i \operatorname{ArcTanh} \left[ \frac{2c(1+d) - be}{\sqrt{b^2 - 4ace}} \right] + i \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ace}} \right] \right) \\
 & \quad \operatorname{Log} \left[ 1 - e^{-2 \left( \operatorname{ArcTanh} \left[ \frac{2c(1+d) - be}{\sqrt{b^2 - 4ace}} \right] + \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ace}} \right] \right)} \right] + \\
 & \quad \pi \operatorname{Log} \left[ \frac{1}{\sqrt{1 - \left( \frac{b}{\sqrt{b^2 - 4ac}} - \frac{2cd}{\sqrt{b^2 - 4ace}} + \frac{2c(d+ex)}{\sqrt{b^2 - 4ace}} \right)^2}} \right] + \\
 & \quad 2i \operatorname{ArcTanh} \left[ \frac{2c(1+d) - be}{\sqrt{b^2 - 4ace}} \right] \operatorname{Log} \left[ \right. \\
 & \quad \quad \left. i \operatorname{Sinh} \left[ \operatorname{ArcTanh} \left[ \frac{2c(1+d) - be}{\sqrt{b^2 - 4ace}} \right] + \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ace}} \right] \right] \right] + \\
 & \quad \left. i \operatorname{PolyLog} \left[ 2, e^{-2 \left( \operatorname{ArcTanh} \left[ \frac{2c(1+d) - be}{\sqrt{b^2 - 4ace}} \right] + \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ace}} \right] \right)} \right] \right) \left. \right) -
 \end{aligned}$$

$$\frac{1}{(b^2 - 4ac) e^2 (-2c - 2cd + be) \sqrt{\frac{(b^2 - 4ac) e^2 - (2c(1+d) - be)^2}{(b^2 - 4ac) e^2}}}$$

$$2c^3 \left( -e^{-\operatorname{ArcTanh} \left[ \frac{2c(1+d) - be}{\sqrt{b^2 - 4ace}} \right]} \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ace}} \right] \right)^2 +$$

$$\frac{1}{\sqrt{b^2 - 4ac} e \sqrt{1 - \frac{(2c(1+d) - be)^2}{(b^2 - 4ac) e^2}}}$$

$$\begin{aligned}
 & i (2 c (1+d) - b e) \left( - \left( -\pi + 2 i \operatorname{ArcTanh} \left[ \frac{2 c (1+d) - b e}{\sqrt{b^2 - 4 a c e}} \right] \right) \right. \\
 & \quad \operatorname{ArcTanh} \left[ \frac{-2 c d + b e + 2 c (d+e x)}{\sqrt{b^2 - 4 a c e}} \right] - \pi \operatorname{Log} \left[ 1 + e^{\frac{2 \operatorname{ArcTanh} \left[ \frac{-2 c d + b e + 2 c (d+e x)}{\sqrt{b^2 - 4 a c e}} \right]}{\sqrt{b^2 - 4 a c e}}} \right] - \\
 & \quad 2 \left( i \operatorname{ArcTanh} \left[ \frac{2 c (1+d) - b e}{\sqrt{b^2 - 4 a c e}} \right] + i \operatorname{ArcTanh} \left[ \frac{-2 c d + b e + 2 c (d+e x)}{\sqrt{b^2 - 4 a c e}} \right] \right) \\
 & \quad \operatorname{Log} \left[ 1 - e^{-2 \left( \operatorname{ArcTanh} \left[ \frac{2 c (1+d) - b e}{\sqrt{b^2 - 4 a c e}} \right] + \operatorname{ArcTanh} \left[ \frac{-2 c d + b e + 2 c (d+e x)}{\sqrt{b^2 - 4 a c e}} \right] \right)} \right] + \\
 & \quad \pi \operatorname{Log} \left[ \frac{1}{\sqrt{1 - \left( \frac{b}{\sqrt{b^2 - 4 a c}} - \frac{2 c d}{\sqrt{b^2 - 4 a c e}} + \frac{2 c (d+e x)}{\sqrt{b^2 - 4 a c e}} \right)^2}} \right] + \\
 & \quad 2 i \operatorname{ArcTanh} \left[ \frac{2 c (1+d) - b e}{\sqrt{b^2 - 4 a c e}} \right] \operatorname{Log} \left[ \right. \\
 & \quad \quad \left. i \operatorname{Sinh} \left[ \operatorname{ArcTanh} \left[ \frac{2 c (1+d) - b e}{\sqrt{b^2 - 4 a c e}} \right] + \operatorname{ArcTanh} \left[ \frac{-2 c d + b e + 2 c (d+e x)}{\sqrt{b^2 - 4 a c e}} \right] \right] \right] + \\
 & \quad \left. i \operatorname{PolyLog} \left[ 2, e^{-2 \left( \operatorname{ArcTanh} \left[ \frac{2 c (1+d) - b e}{\sqrt{b^2 - 4 a c e}} \right] + \operatorname{ArcTanh} \left[ \frac{-2 c d + b e + 2 c (d+e x)}{\sqrt{b^2 - 4 a c e}} \right] \right)} \right] \right) \right) - \\
 & \quad \frac{1}{(b^2 - 4 a c) e^2 (-2 c - 2 c d + b e) \sqrt{\frac{(b^2 - 4 a c) e^2 - (2 c (1+d) - b e)^2}{(b^2 - 4 a c) e^2}}} \\
 & \quad 4 c^3 d \left( - e^{-\operatorname{ArcTanh} \left[ \frac{2 c (1+d) - b e}{\sqrt{b^2 - 4 a c e}} \right]} \operatorname{ArcTanh} \left[ \frac{-2 c d + b e + 2 c (d+e x)}{\sqrt{b^2 - 4 a c e}} \right] \right)^2 + \\
 & \quad \frac{1}{\sqrt{b^2 - 4 a c e} \sqrt{1 - \frac{(2 c (1+d) - b e)^2}{(b^2 - 4 a c) e^2}}}
 \end{aligned}$$

$$\begin{aligned}
 & i (2c(1+d) - be) \left( -\left( -\pi + 2i \operatorname{ArcTanh} \left[ \frac{2c(1+d) - be}{\sqrt{b^2 - 4ac}e} \right] \right) \right. \\
 & \quad \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac}e} \right] - \pi \operatorname{Log} \left[ 1 + e^{\frac{2 \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac}e} \right]}{}} \right] - \\
 & \quad 2 \left( i \operatorname{ArcTanh} \left[ \frac{2c(1+d) - be}{\sqrt{b^2 - 4ac}e} \right] + i \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac}e} \right] \right) \\
 & \quad \operatorname{Log} \left[ 1 - e^{-2 \left( \operatorname{ArcTanh} \left[ \frac{2c(1+d) - be}{\sqrt{b^2 - 4ac}e} \right] + \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac}e} \right] \right)} \right] + \\
 & \quad \pi \operatorname{Log} \left[ \frac{1}{\sqrt{1 - \left( \frac{b}{\sqrt{b^2 - 4ac}} - \frac{2cd}{\sqrt{b^2 - 4ac}e} + \frac{2c(d+ex)}{\sqrt{b^2 - 4ac}e} \right)^2}} \right] + \\
 & \quad 2i \operatorname{ArcTanh} \left[ \frac{2c(1+d) - be}{\sqrt{b^2 - 4ac}e} \right] \operatorname{Log} \left[ \right. \\
 & \quad \quad \left. i \operatorname{Sinh} \left[ \operatorname{ArcTanh} \left[ \frac{2c(1+d) - be}{\sqrt{b^2 - 4ac}e} \right] + \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac}e} \right] \right] \right] + \\
 & \quad \left. i \operatorname{PolyLog} \left[ 2, e^{-2 \left( \operatorname{ArcTanh} \left[ \frac{2c(1+d) - be}{\sqrt{b^2 - 4ac}e} \right] + \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac}e} \right] \right)} \right] \right) \right) -
 \end{aligned}$$

$$\frac{1}{(b^2 - 4ac) e^2 (-2c - 2cd + be) \sqrt{\frac{(b^2 - 4ac) e^2 - (2c(1+d) - be)^2}{(b^2 - 4ac) e^2}}}$$

$$2c^3 d^2 \left( -e^{-\operatorname{ArcTanh} \left[ \frac{2c(1+d) - be}{\sqrt{b^2 - 4ac}e} \right]} \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac}e} \right] \right)^2 +$$

$$\frac{1}{\sqrt{b^2 - 4ac} e \sqrt{1 - \frac{(2c(1+d) - be)^2}{(b^2 - 4ac) e^2}}}$$

$$\begin{aligned}
 & i (2 c (1+d) - b e) \left( - \left( -\pi + 2 i \operatorname{ArcTanh} \left[ \frac{2 c (1+d) - b e}{\sqrt{b^2 - 4 a c e}} \right] \right) \right. \\
 & \operatorname{ArcTanh} \left[ \frac{-2 c d + b e + 2 c (d+e x)}{\sqrt{b^2 - 4 a c e}} \right] - \pi \operatorname{Log} \left[ 1 + e^{\frac{2 \operatorname{ArcTanh} \left[ \frac{-2 c d + b e + 2 c (d+e x)}{\sqrt{b^2 - 4 a c e}} \right]}{\sqrt{b^2 - 4 a c e}}} \right] - \\
 & 2 \left( i \operatorname{ArcTanh} \left[ \frac{2 c (1+d) - b e}{\sqrt{b^2 - 4 a c e}} \right] + i \operatorname{ArcTanh} \left[ \frac{-2 c d + b e + 2 c (d+e x)}{\sqrt{b^2 - 4 a c e}} \right] \right) \\
 & \operatorname{Log} \left[ 1 - e^{-2 \left( \operatorname{ArcTanh} \left[ \frac{2 c (1+d) - b e}{\sqrt{b^2 - 4 a c e}} \right] + \operatorname{ArcTanh} \left[ \frac{-2 c d + b e + 2 c (d+e x)}{\sqrt{b^2 - 4 a c e}} \right] \right)} \right] + \\
 & \pi \operatorname{Log} \left[ \frac{1}{\sqrt{1 - \left( \frac{b}{\sqrt{b^2 - 4 a c}} - \frac{2 c d}{\sqrt{b^2 - 4 a c e}} + \frac{2 c (d+e x)}{\sqrt{b^2 - 4 a c e}} \right)^2}} \right] + \\
 & 2 i \operatorname{ArcTanh} \left[ \frac{2 c (1+d) - b e}{\sqrt{b^2 - 4 a c e}} \right] \operatorname{Log} \left[ \right. \\
 & \left. i \operatorname{Sinh} \left[ \operatorname{ArcTanh} \left[ \frac{2 c (1+d) - b e}{\sqrt{b^2 - 4 a c e}} \right] + \operatorname{ArcTanh} \left[ \frac{-2 c d + b e + 2 c (d+e x)}{\sqrt{b^2 - 4 a c e}} \right] \right] \right] + \\
 & \left. i \operatorname{PolyLog} \left[ 2, e^{-2 \left( \operatorname{ArcTanh} \left[ \frac{2 c (1+d) - b e}{\sqrt{b^2 - 4 a c e}} \right] + \operatorname{ArcTanh} \left[ \frac{-2 c d + b e + 2 c (d+e x)}{\sqrt{b^2 - 4 a c e}} \right] \right)} \right] \right) \right) + \\
 & \frac{1}{(b^2 - 4 a c) e (-2 c - 2 c d + b e) \sqrt{\frac{(b^2 - 4 a c) e^2 - (2 c (1+d) - b e)^2}{(b^2 - 4 a c) e^2}}} \\
 & 2 b c^2 \left( - e^{-\operatorname{ArcTanh} \left[ \frac{2 c (1+d) - b e}{\sqrt{b^2 - 4 a c e}} \right]} \operatorname{ArcTanh} \left[ \frac{-2 c d + b e + 2 c (d+e x)}{\sqrt{b^2 - 4 a c e}} \right] \right)^2 + \\
 & \frac{1}{\sqrt{b^2 - 4 a c} e \sqrt{1 - \frac{(2 c (1+d) - b e)^2}{(b^2 - 4 a c) e^2}}}
 \end{aligned}$$

$$\begin{aligned}
 & i (2c(1+d) - be) \left( -\left( -\pi + 2i \operatorname{ArcTanh} \left[ \frac{2c(1+d) - be}{\sqrt{b^2 - 4ace}} \right] \right) \right. \\
 & \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ace}} \right] - \pi \operatorname{Log} \left[ 1 + e^{\frac{2 \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ace}} \right]}{\sqrt{b^2 - 4ace}}} \right] - \\
 & 2 \left( i \operatorname{ArcTanh} \left[ \frac{2c(1+d) - be}{\sqrt{b^2 - 4ace}} \right] + i \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ace}} \right] \right) \\
 & \operatorname{Log} \left[ 1 - e^{-2 \left( \operatorname{ArcTanh} \left[ \frac{2c(1+d) - be}{\sqrt{b^2 - 4ace}} \right] + \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ace}} \right] \right)} \right] + \\
 & \pi \operatorname{Log} \left[ \frac{1}{\sqrt{1 - \left( \frac{b}{\sqrt{b^2 - 4ac}} - \frac{2cd}{\sqrt{b^2 - 4ace}} + \frac{2c(d+ex)}{\sqrt{b^2 - 4ace}} \right)^2}} \right] + \\
 & 2i \operatorname{ArcTanh} \left[ \frac{2c(1+d) - be}{\sqrt{b^2 - 4ace}} \right] \operatorname{Log} \left[ \right. \\
 & \left. i \operatorname{Sinh} \left[ \operatorname{ArcTanh} \left[ \frac{2c(1+d) - be}{\sqrt{b^2 - 4ace}} \right] + \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ace}} \right] \right] \right] + \\
 & \left. i \operatorname{PolyLog} \left[ 2, e^{-2 \left( \operatorname{ArcTanh} \left[ \frac{2c(1+d) - be}{\sqrt{b^2 - 4ace}} \right] + \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ace}} \right] \right)} \right] \right) \right) +
 \end{aligned}$$

$$\frac{1}{(b^2 - 4ac)e(-2c - 2cd + be) \sqrt{\frac{(b^2 - 4ac)e^2 - (2c(1+d) - be)^2}{(b^2 - 4ac)e^2}}}$$

$$2bc^2d \left( -e^{\frac{-\operatorname{ArcTanh} \left[ \frac{2c(1+d) - be}{\sqrt{b^2 - 4ace}} \right]}{\sqrt{b^2 - 4ace}}} \operatorname{ArcTanh} \left[ \frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ace}} \right]^2 + \right.$$

$$\left. \frac{1}{\sqrt{b^2 - 4ac}e \sqrt{1 - \frac{(2c(1+d) - be)^2}{(b^2 - 4ac)e^2}}} \right)$$

$$\begin{aligned}
 & i (2 c (1+d) - b e) \left( -\left( -\pi + 2 i \operatorname{ArcTanh} \left[ \frac{2 c (1+d) - b e}{\sqrt{b^2 - 4 a c e}} \right] \right) \right. \\
 & \quad \operatorname{ArcTanh} \left[ \frac{-2 c d + b e + 2 c (d+e x)}{\sqrt{b^2 - 4 a c e}} \right] - \pi \operatorname{Log} \left[ 1 + e^{2 \operatorname{ArcTanh} \left[ \frac{-2 c d + b e + 2 c (d+e x)}{\sqrt{b^2 - 4 a c e}} \right]} \right] - \\
 & \quad 2 \left( i \operatorname{ArcTanh} \left[ \frac{2 c (1+d) - b e}{\sqrt{b^2 - 4 a c e}} \right] + i \operatorname{ArcTanh} \left[ \frac{-2 c d + b e + 2 c (d+e x)}{\sqrt{b^2 - 4 a c e}} \right] \right) \\
 & \quad \operatorname{Log} \left[ 1 - e^{-2 \left( \operatorname{ArcTanh} \left[ \frac{2 c (1+d) - b e}{\sqrt{b^2 - 4 a c e}} \right] + \operatorname{ArcTanh} \left[ \frac{-2 c d + b e + 2 c (d+e x)}{\sqrt{b^2 - 4 a c e}} \right] \right)} \right] + \\
 & \quad \pi \operatorname{Log} \left[ \frac{1}{\sqrt{1 - \left( \frac{b}{\sqrt{b^2 - 4 a c}} - \frac{2 c d}{\sqrt{b^2 - 4 a c e}} + \frac{2 c (d+e x)}{\sqrt{b^2 - 4 a c e}} \right)^2}} \right] + \\
 & \quad 2 i \operatorname{ArcTanh} \left[ \frac{2 c (1+d) - b e}{\sqrt{b^2 - 4 a c e}} \right] \operatorname{Log} \left[ \right. \\
 & \quad \quad \left. i \operatorname{Sinh} \left[ \operatorname{ArcTanh} \left[ \frac{2 c (1+d) - b e}{\sqrt{b^2 - 4 a c e}} \right] + \operatorname{ArcTanh} \left[ \frac{-2 c d + b e + 2 c (d+e x)}{\sqrt{b^2 - 4 a c e}} \right] \right] \right] + \\
 & \quad \left. i \operatorname{PolyLog} \left[ 2, e^{-2 \left( \operatorname{ArcTanh} \left[ \frac{2 c (1+d) - b e}{\sqrt{b^2 - 4 a c e}} \right] + \operatorname{ArcTanh} \left[ \frac{-2 c d + b e + 2 c (d+e x)}{\sqrt{b^2 - 4 a c e}} \right] \right)} \right] \right) \right)
 \end{aligned}$$

Problem 95: Result unnecessarily involves higher level functions.

$$\int \frac{\operatorname{ArcCoth} [a x^n]}{x} dx$$

Optimal (type 4, 38 leaves, 2 steps):

$$\frac{\operatorname{PolyLog} \left[ 2, -\frac{x^n}{a} \right]}{2 n} - \frac{\operatorname{PolyLog} \left[ 2, \frac{x^n}{a} \right]}{2 n}$$

Result (type 5, 52 leaves):

$$\frac{a x^n \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{2}, \frac{1}{2}, 1 \right\}, \left\{ \frac{3}{2}, \frac{3}{2} \right\}, a^2 x^{2 n} \right]}{n} + \left( \operatorname{ArcCoth} [a x^n] - \operatorname{ArcTanh} [a x^n] \right) \operatorname{Log} [x]$$

Problem 100: Result unnecessarily involves complex numbers and more than



twice size of optimal antiderivative.

$$\int \frac{\text{ArcCoth}[1+x]}{2+2x} dx$$

Optimal (type 4, 25 leaves, 3 steps):

$$\frac{1}{4} \text{PolyLog}\left[2, -\frac{1}{1+x}\right] - \frac{1}{4} \text{PolyLog}\left[2, \frac{1}{1+x}\right]$$

Result (type 4, 227 leaves):

$$\begin{aligned} & \frac{1}{16} \left( -\pi^2 + 4 i \pi \text{ArcTanh}[1+x] + 8 \text{ArcTanh}[1+x]^2 + 8 \text{ArcTanh}[1+x] \text{Log}\left[1 - e^{-2 \text{ArcTanh}[1+x]}\right] - \right. \\ & 4 i \pi \text{Log}\left[1 + e^{2 \text{ArcTanh}[1+x]}\right] - 8 \text{ArcTanh}[1+x] \text{Log}\left[1 + e^{2 \text{ArcTanh}[1+x]}\right] + \\ & 8 \text{ArcCoth}[1+x] \text{Log}[1+x] - 8 \text{ArcTanh}[1+x] \text{Log}[1+x] - 8 \text{ArcTanh}[1+x] \text{Log}\left[\frac{1}{\sqrt{-x(2+x)}}\right] + \\ & 4 i \pi \text{Log}\left[\frac{2}{\sqrt{-x(2+x)}}\right] + 8 \text{ArcTanh}[1+x] \text{Log}\left[\frac{2}{\sqrt{-x(2+x)}}\right] + \\ & 8 \text{ArcTanh}[1+x] \text{Log}\left[\frac{i(1+x)}{\sqrt{-x(2+x)}}\right] - 8 \text{ArcTanh}[1+x] \text{Log}\left[\frac{2 i(1+x)}{\sqrt{-x(2+x)}}\right] - \\ & \left. 4 \text{PolyLog}\left[2, e^{-2 \text{ArcTanh}[1+x]}\right] - 4 \text{PolyLog}\left[2, -e^{2 \text{ArcTanh}[1+x]}\right] \right) \end{aligned}$$

**Problem 101:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcCoth}[a+bx]}{\frac{ad}{b} + dx} dx$$

Optimal (type 4, 35 leaves, 3 steps):

$$\frac{\text{PolyLog}\left[2, -\frac{1}{a+bx}\right]}{2d} - \frac{\text{PolyLog}\left[2, \frac{1}{a+bx}\right]}{2d}$$

Result (type 4, 291 leaves):

$$\begin{aligned}
 & -\frac{1}{8d} \left( \pi^2 - 4i\pi \operatorname{ArcTanh}[a+bx] - 8 \operatorname{ArcTanh}[a+bx]^2 - \right. \\
 & 8 \operatorname{ArcTanh}[a+bx] \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcTanh}[a+bx]}\right] + 4i\pi \operatorname{Log}\left[1 + e^{2 \operatorname{ArcTanh}[a+bx]}\right] + \\
 & 8 \operatorname{ArcTanh}[a+bx] \operatorname{Log}\left[1 + e^{2 \operatorname{ArcTanh}[a+bx]}\right] - 8 \operatorname{ArcCoth}[a+bx] \operatorname{Log}[a+bx] + \\
 & 8 \operatorname{ArcTanh}[a+bx] \operatorname{Log}[a+bx] + 8 \operatorname{ArcTanh}[a+bx] \operatorname{Log}\left[\frac{1}{\sqrt{1 - (a+bx)^2}}\right] - \\
 & 4i\pi \operatorname{Log}\left[\frac{2}{\sqrt{1 - (a+bx)^2}}\right] - 8 \operatorname{ArcTanh}[a+bx] \operatorname{Log}\left[\frac{2}{\sqrt{1 - (a+bx)^2}}\right] - \\
 & 8 \operatorname{ArcTanh}[a+bx] \operatorname{Log}\left[\frac{i(a+bx)}{\sqrt{1 - (a+bx)^2}}\right] + 8 \operatorname{ArcTanh}[a+bx] \operatorname{Log}\left[\frac{2i(a+bx)}{\sqrt{1 - (a+bx)^2}}\right] + \\
 & \left. 4 \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcTanh}[a+bx]}\right] + 4 \operatorname{PolyLog}\left[2, -e^{2 \operatorname{ArcTanh}[a+bx]}\right] \right)
 \end{aligned}$$

**Problem 106: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{a + b \operatorname{ArcCoth}[c + dx]}{e + fx} dx$$

Optimal (type 4, 130 leaves, 5 steps):

$$\begin{aligned}
 & -\frac{(a + b \operatorname{ArcCoth}[c + dx]) \operatorname{Log}\left[\frac{2}{1+c+dx}\right]}{f} + \frac{(a + b \operatorname{ArcCoth}[c + dx]) \operatorname{Log}\left[\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right]}{f} + \\
 & \frac{b \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+c+dx}\right]}{2f} - \frac{b \operatorname{PolyLog}\left[2, 1 - \frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right]}{2f}
 \end{aligned}$$

Result (type 4, 352 leaves):

$$\begin{aligned}
 & \frac{1}{f} \left( a \operatorname{Log}[e + f x] + b (\operatorname{ArcCoth}[c + d x] - \operatorname{ArcTanh}[c + d x]) \operatorname{Log}[e + f x] + b \operatorname{ArcTanh}[c + d x] \right. \\
 & \quad \left. - \operatorname{Log}\left[\frac{1}{\sqrt{1 - (c + d x)^2}}\right] + \operatorname{Log}\left[i \operatorname{Sinh}\left[\operatorname{ArcTanh}\left[\frac{d e - c f}{f}\right] + \operatorname{ArcTanh}[c + d x]\right]\right] \right) - \\
 & \frac{1}{2} i b \left( -\frac{1}{4} i (\pi - 2 i \operatorname{ArcTanh}[c + d x])^2 + i \left( \operatorname{ArcTanh}\left[\frac{d e - c f}{f}\right] + \operatorname{ArcTanh}[c + d x] \right)^2 + \right. \\
 & \quad (\pi - 2 i \operatorname{ArcTanh}[c + d x]) \operatorname{Log}\left[1 + e^{2 \operatorname{ArcTanh}[c + d x]}\right] + \\
 & \quad 2 i \left( \operatorname{ArcTanh}\left[\frac{d e - c f}{f}\right] + \operatorname{ArcTanh}[c + d x] \right) \operatorname{Log}\left[1 - e^{-2 (\operatorname{ArcTanh}\left[\frac{d e - c f}{f}\right] + \operatorname{ArcTanh}[c + d x])}\right] - \\
 & \quad (\pi - 2 i \operatorname{ArcTanh}[c + d x]) \operatorname{Log}\left[\frac{2}{\sqrt{1 - (c + d x)^2}}\right] - 2 i \left( \operatorname{ArcTanh}\left[\frac{d e - c f}{f}\right] + \operatorname{ArcTanh}[c + d x] \right) \\
 & \quad \operatorname{Log}\left[2 i \operatorname{Sinh}\left[\operatorname{ArcTanh}\left[\frac{d e - c f}{f}\right] + \operatorname{ArcTanh}[c + d x]\right]\right] - \\
 & \quad \left. i \operatorname{PolyLog}\left[2, -e^{2 \operatorname{ArcTanh}[c + d x]}\right] - i \operatorname{PolyLog}\left[2, e^{-2 (\operatorname{ArcTanh}\left[\frac{d e - c f}{f}\right] + \operatorname{ArcTanh}[c + d x])}\right] \right) \Big)
 \end{aligned}$$

### Problem 109: Result more than twice size of optimal antiderivative.

$$\int (e + f x)^2 (a + b \operatorname{ArcCoth}[c + d x])^2 dx$$

Optimal (type 4, 374 leaves, 16 steps):

$$\begin{aligned}
 & \frac{b^2 f^2 x}{3 d^2} + \frac{2 a b f (d e - c f) x}{d^2} + \\
 & \frac{2 b^2 f (d e - c f) (c + d x) \operatorname{ArcCoth}[c + d x]}{d^3} + \frac{b f^2 (c + d x)^2 (a + b \operatorname{ArcCoth}[c + d x])}{3 d^3} - \\
 & \frac{(d e - c f) (d^2 e^2 - 2 c d e f + (3 + c^2) f^2) (a + b \operatorname{ArcCoth}[c + d x])^2}{3 d^3 f} + \\
 & \frac{(3 d^2 e^2 - 6 c d e f + (1 + 3 c^2) f^2) (a + b \operatorname{ArcCoth}[c + d x])^2}{3 d^3} + \\
 & \frac{(e + f x)^3 (a + b \operatorname{ArcCoth}[c + d x])^2}{3 f} - \frac{b^2 f^2 \operatorname{ArcTanh}[c + d x]}{3 d^3} - \frac{1}{3 d^3} \\
 & 2 b (3 d^2 e^2 - 6 c d e f + (1 + 3 c^2) f^2) (a + b \operatorname{ArcCoth}[c + d x]) \operatorname{Log}\left[\frac{2}{1 - c - d x}\right] + \\
 & \frac{b^2 f (d e - c f) \operatorname{Log}[1 - (c + d x)^2]}{d^3} - \frac{b^2 (3 d^2 e^2 - 6 c d e f + (1 + 3 c^2) f^2) \operatorname{PolyLog}\left[2, -\frac{1 + c + d x}{1 - c - d x}\right]}{3 d^3}
 \end{aligned}$$

Result (type 4, 1054 leaves):

$$a^2 e^2 x + a^2 e f x^2 + \frac{1}{3} a^2 f^2 x^3 + \frac{1}{3} a b \left( 2 x (3 e^2 + 3 e f x + f^2 x^2) \operatorname{ArcCoth}[c + d x] + \frac{1}{d^3} \right)$$

$$\begin{aligned}
 & \left( d f x (6 d e - 4 c f + d f x) - (-1 + c) (3 d^2 e^2 - 3 (-1 + c) d e f + (-1 + c)^2 f^2) \operatorname{Log}[1 - c - d x] + \right. \\
 & \quad \left. (1 + c) (3 d^2 e^2 - 3 (1 + c) d e f + (1 + c)^2 f^2) \operatorname{Log}[1 + c + d x] \right) + (b^2 e^2 (1 - (c + d x)^2) \\
 & \quad (\operatorname{ArcCoth}[c + d x] (\operatorname{ArcCoth}[c + d x] - (c + d x) \operatorname{ArcCoth}[c + d x] + 2 \operatorname{Log}[1 - e^{-2 \operatorname{ArcCoth}[c + d x]}]) - \\
 & \quad \operatorname{PolyLog}[2, e^{-2 \operatorname{ArcCoth}[c + d x]}])) / \left( d (c + d x)^2 \left( 1 - \frac{1}{(c + d x)^2} \right) \right) - \\
 & \left( b^2 e f (1 - (c + d x)^2) \left( 2 c \operatorname{ArcCoth}[c + d x]^2 + (c + d x)^2 \left( 1 - \frac{1}{(c + d x)^2} \right) \operatorname{ArcCoth}[c + d x]^2 - \right. \right. \\
 & \quad \left. \left. 2 (c + d x) \operatorname{ArcCoth}[c + d x] (-1 + c \operatorname{ArcCoth}[c + d x]) + 4 c \operatorname{ArcCoth}[c + d x] \operatorname{Log}\left[ \frac{1 - e^{-2 \operatorname{ArcCoth}[c + d x]}}{(c + d x) \sqrt{1 - \frac{1}{(c + d x)^2}}} \right] - 2 c \operatorname{PolyLog}[2, e^{-2 \operatorname{ArcCoth}[c + d x]}] \right) \right) / \\
 & \left( d^2 (c + d x)^2 \left( 1 - \frac{1}{(c + d x)^2} \right) \right) - \frac{1}{12 d^3} b^2 f^2 (c + d x) \sqrt{1 - \frac{1}{(c + d x)^2}} (1 - (c + d x)^2) \\
 & \left( \frac{4 \operatorname{ArcCoth}[c + d x]}{(c + d x) \sqrt{1 - \frac{1}{(c + d x)^2}}} + \frac{3 \operatorname{ArcCoth}[c + d x]^2}{(c + d x) \sqrt{1 - \frac{1}{(c + d x)^2}}} - \right. \\
 & \quad \frac{12 c \operatorname{ArcCoth}[c + d x]^2}{(c + d x) \sqrt{1 - \frac{1}{(c + d x)^2}}} + \frac{9 c^2 \operatorname{ArcCoth}[c + d x]^2}{(c + d x) \sqrt{1 - \frac{1}{(c + d x)^2}}} + \frac{1}{\sqrt{1 - \frac{1}{(c + d x)^2}}} \\
 & \quad (-1 + 6 c \operatorname{ArcCoth}[c + d x] + 3 \operatorname{ArcCoth}[c + d x]^2 - 3 c^2 \operatorname{ArcCoth}[c + d x]^2) + \\
 & \quad \operatorname{Cosh}[3 \operatorname{ArcCoth}[c + d x]] - 6 c \operatorname{ArcCoth}[c + d x] \operatorname{Cosh}[3 \operatorname{ArcCoth}[c + d x]] + \\
 & \quad \operatorname{ArcCoth}[c + d x]^2 \operatorname{Cosh}[3 \operatorname{ArcCoth}[c + d x]] + 3 c^2 \operatorname{ArcCoth}[c + d x]^2 \operatorname{Cosh}[3 \operatorname{ArcCoth}[c + d x]] + \\
 & \quad \frac{6 \operatorname{ArcCoth}[c + d x] \operatorname{Log}[1 - e^{-2 \operatorname{ArcCoth}[c + d x]}]}{(c + d x) \sqrt{1 - \frac{1}{(c + d x)^2}}} + \frac{18 c^2 \operatorname{ArcCoth}[c + d x] \operatorname{Log}[1 - e^{-2 \operatorname{ArcCoth}[c + d x]}]}{(c + d x) \sqrt{1 - \frac{1}{(c + d x)^2}}} - \\
 & \quad \frac{18 c \operatorname{Log}\left[ \frac{1}{(c + d x) \sqrt{1 - \frac{1}{(c + d x)^2}}} \right]}{(c + d x) \sqrt{1 - \frac{1}{(c + d x)^2}}} + \frac{4 (1 + 3 c^2) \operatorname{PolyLog}[2, e^{-2 \operatorname{ArcCoth}[c + d x]}]}{(c + d x)^3 \left( 1 - \frac{1}{(c + d x)^2} \right)^{3/2}} - \\
 & \quad \operatorname{ArcCoth}[c + d x]^2 \operatorname{Sinh}[3 \operatorname{ArcCoth}[c + d x]] - 3 c^2 \operatorname{ArcCoth}[c + d x]^2 \operatorname{Sinh}[3 \operatorname{ArcCoth}[c + d x]] - \\
 & \quad 2 \operatorname{ArcCoth}[c + d x] \operatorname{Log}[1 - e^{-2 \operatorname{ArcCoth}[c + d x]}] \operatorname{Sinh}[3 \operatorname{ArcCoth}[c + d x]] - \\
 & \quad 6 c^2 \operatorname{ArcCoth}[c + d x] \operatorname{Log}[1 - e^{-2 \operatorname{ArcCoth}[c + d x]}] \operatorname{Sinh}[3 \operatorname{ArcCoth}[c + d x]] +
 \end{aligned}$$

$$\left. 6 c \operatorname{Log} \left[ \frac{1}{(c+d x) \sqrt{1-\frac{1}{(c+d x)^2}}} \right] \operatorname{Sinh} [3 \operatorname{ArcCoth} [c+d x]] \right\}$$

**Problem 112: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \operatorname{ArcCoth} [c+d x])^2}{e+f x} dx$$

Optimal (type 4, 214 leaves, 2 steps):

$$\begin{aligned} & -\frac{(a+b \operatorname{ArcCoth} [c+d x])^2 \operatorname{Log} \left[ \frac{2}{1+c+d x} \right]}{f} + \frac{(a+b \operatorname{ArcCoth} [c+d x])^2 \operatorname{Log} \left[ \frac{2 d (e+f x)}{(d e+f-c f)(1+c+d x)} \right]}{f} + \\ & \frac{b(a+b \operatorname{ArcCoth} [c+d x]) \operatorname{PolyLog} [2, 1-\frac{2}{1+c+d x}]}{f} - \\ & \frac{b(a+b \operatorname{ArcCoth} [c+d x]) \operatorname{PolyLog} [2, 1-\frac{2 d (e+f x)}{(d e+f-c f)(1+c+d x)}]}{f} + \\ & \frac{b^2 \operatorname{PolyLog} [3, 1-\frac{2}{1+c+d x}]}{2 f} - \frac{b^2 \operatorname{PolyLog} [3, 1-\frac{2 d (e+f x)}{(d e+f-c f)(1+c+d x)}]}{2 f} \end{aligned}$$

Result (type 4, 1640 leaves):

$$\begin{aligned} & \frac{a^2 \operatorname{Log} [e+f x]}{f} + 2 a b \left( \frac{(\operatorname{ArcCoth} [c+d x] - \operatorname{ArcTanh} [c+d x]) \operatorname{Log} [e+f x]}{f} - \frac{1}{f} i \left( i \operatorname{ArcTanh} [c+d x] \right. \right. \\ & \left. \left. \left( -\operatorname{Log} \left[ \frac{1}{\sqrt{1-(c+d x)^2}} \right] + \operatorname{Log} [i \operatorname{Sinh} [\operatorname{ArcTanh} [\frac{d e-c f}{f}] + \operatorname{ArcTanh} [c+d x]]] \right) \right) + \right. \\ & \left. \frac{1}{2} \left( -i \left( i \operatorname{ArcTanh} [\frac{d e-c f}{f}] + i \operatorname{ArcTanh} [c+d x] \right)^2 - \frac{1}{4} i (\pi - 2 i \operatorname{ArcTanh} [c+d x])^2 + \right. \right. \\ & \left. \left. 2 \left( i \operatorname{ArcTanh} [\frac{d e-c f}{f}] + i \operatorname{ArcTanh} [c+d x] \right) \operatorname{Log} [1 - e^{2 i (i \operatorname{ArcTanh} [\frac{d e-c f}{f}] + i \operatorname{ArcTanh} [c+d x])}] \right) + \right. \\ & \left. (\pi - 2 i \operatorname{ArcTanh} [c+d x]) \operatorname{Log} [1 - e^{i (\pi - 2 i \operatorname{ArcTanh} [c+d x])}] - (\pi - 2 i \operatorname{ArcTanh} [c+d x]) \operatorname{Log} [ \right. \\ & \left. 2 \operatorname{Sin} [\frac{1}{2} (\pi - 2 i \operatorname{ArcTanh} [c+d x])] \right] - 2 \left( i \operatorname{ArcTanh} [\frac{d e-c f}{f}] + i \operatorname{ArcTanh} [c+d x] \right) \\ & \left. \operatorname{Log} [2 i \operatorname{Sinh} [\operatorname{ArcTanh} [\frac{d e-c f}{f}] + \operatorname{ArcTanh} [c+d x]]] - i \right. \\ & \left. \operatorname{PolyLog} [2, e^{2 i (i \operatorname{ArcTanh} [\frac{d e-c f}{f}] + i \operatorname{ArcTanh} [c+d x])}] - i \operatorname{PolyLog} [2, e^{i (\pi - 2 i \operatorname{ArcTanh} [c+d x])}] \right) \left. \right) - \\ & \frac{1}{d (c+d x)^2 (e+f x) \left( 1 - \frac{1}{(c+d x)^2} \right)} b^2 (d e-c f+f (c+d x)) \end{aligned}$$

$$\begin{aligned}
 & \left( 1 - (c + dx)^2 \right) \\
 & \left( -\frac{1}{24 f^2} \left( i f \pi^3 - 8 d e \operatorname{ArcCoth}[c + dx]^3 - 8 f \operatorname{ArcCoth}[c + dx]^3 + \right. \right. \\
 & \quad \left. \left. 8 c f \operatorname{ArcCoth}[c + dx]^3 + 24 f \operatorname{ArcCoth}[c + dx]^2 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcCoth}[c + dx]}\right] + \right. \right. \\
 & \quad \left. \left. 24 f \operatorname{ArcCoth}[c + dx] \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcCoth}[c + dx]}\right] - 12 f \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcCoth}[c + dx]}\right] \right) + \right. \\
 & \quad \left. \frac{1}{6 f^2 (d e + f - c f) (d e - (1 + c) f)} (-d e - f + c f) (-d e + f + c f) \right. \\
 & \left. \left( 2 d e \operatorname{ArcCoth}[c + dx]^3 - 6 f \operatorname{ArcCoth}[c + dx]^3 - 2 c f \operatorname{ArcCoth}[c + dx]^3 - \right. \right. \\
 & \quad \left. \left. 4 d e e^{-\operatorname{ArcTanh}\left[\frac{f}{d e - c f}\right]} \sqrt{\frac{d^2 e^2 - 2 c d e f + (-1 + c^2) f^2}{(d e - c f)^2}} \operatorname{ArcCoth}[c + dx]^3 + 4 c e^{-\operatorname{ArcTanh}\left[\frac{f}{d e - c f}\right]} \right. \right. \\
 & \quad \left. \left. f \sqrt{\frac{d^2 e^2 - 2 c d e f + (-1 + c^2) f^2}{(d e - c f)^2}} \operatorname{ArcCoth}[c + dx]^3 + 6 i f \pi \operatorname{ArcCoth}[c + dx] \operatorname{Log}[2] - \right. \right. \\
 & \quad \left. \left. f \operatorname{ArcCoth}[c + dx]^2 \operatorname{Log}[64] - 6 i f \pi \operatorname{ArcCoth}[c + dx] \operatorname{Log}\left[e^{-\operatorname{ArcCoth}[c + dx]} + e^{\operatorname{ArcCoth}[c + dx]}\right] + \right. \right. \\
 & \quad \left. \left. 6 f \operatorname{ArcCoth}[c + dx]^2 \operatorname{Log}\left[1 - e^{\operatorname{ArcCoth}[c + dx] + \operatorname{ArcTanh}\left[\frac{f}{d e - c f}\right]}\right] + \right. \right. \\
 & \quad \left. \left. 6 f \operatorname{ArcCoth}[c + dx]^2 \operatorname{Log}\left[1 + e^{\operatorname{ArcCoth}[c + dx] + \operatorname{ArcTanh}\left[\frac{f}{d e - c f}\right]}\right] + 6 f \operatorname{ArcCoth}[c + dx]^2 \right. \right. \\
 & \quad \left. \left. \operatorname{Log}\left[1 - e^{2 \left(\operatorname{ArcCoth}[c + dx] + \operatorname{ArcTanh}\left[\frac{f}{d e - c f}\right]\right)}\right] + 12 f \operatorname{ArcCoth}[c + dx] \operatorname{ArcTanh}\left[\frac{f}{d e - c f}\right] \right. \right. \\
 & \quad \left. \left. \operatorname{Log}\left[\frac{1}{2} i e^{-\operatorname{ArcCoth}[c + dx] - \operatorname{ArcTanh}\left[\frac{f}{d e - c f}\right]} \left(-1 + e^{2 \left(\operatorname{ArcCoth}[c + dx] + \operatorname{ArcTanh}\left[\frac{f}{d e - c f}\right]\right)}\right)\right] + \right. \right. \\
 & \quad \left. \left. 6 f \operatorname{ArcCoth}[c + dx]^2 \operatorname{Log}\left[-e^{-\operatorname{ArcCoth}[c + dx]} (d e (-1 + e^{2 \operatorname{ArcCoth}[c + dx]}) + (1 + c + \right. \right. \right. \right. \\
 & \quad \left. \left. \left. e^{2 \operatorname{ArcCoth}[c + dx]} - c e^{2 \operatorname{ArcCoth}[c + dx]}) f\right] - 6 f \operatorname{ArcCoth}[c + dx]^2 \operatorname{Log}\left[\frac{1}{d e - (1 + c) f} \right. \right. \right. \\
 & \quad \left. \left. \left. (-d e (-1 + e^{2 \operatorname{ArcCoth}[c + dx]}) + (-1 - e^{2 \operatorname{ArcCoth}[c + dx]} + c (-1 + e^{2 \operatorname{ArcCoth}[c + dx]}) f)\right] + \right. \right. \\
 & \quad \left. \left. 6 i f \pi \operatorname{ArcCoth}[c + dx] \operatorname{Log}\left[\frac{1}{\sqrt{1 - \frac{1}{(c + dx)^2}}}\right] - 6 f \operatorname{ArcCoth}[c + dx]^2 \right. \right. \\
 & \quad \left. \left. \operatorname{Log}\left[-\frac{f}{\sqrt{1 - \frac{1}{(c + dx)^2}}} - \frac{d e}{(c + dx) \sqrt{1 - \frac{1}{(c + dx)^2}}} + \frac{c f}{(c + dx) \sqrt{1 - \frac{1}{(c + dx)^2}}}\right] - 12 f \operatorname{ArcCoth}[c + dx] \operatorname{ArcTanh}\left[\frac{f}{d e - c f}\right] \operatorname{Log}\left[i \operatorname{Sinh}\left[\operatorname{ArcCoth}[c + dx] + \operatorname{ArcTanh}\left[\frac{f}{d e - c f}\right]\right]\right] + \right. \\
 & \quad \left. \left. 12 f \operatorname{ArcCoth}[c + dx] \operatorname{PolyLog}\left[2, -e^{\operatorname{ArcCoth}[c + dx] + \operatorname{ArcTanh}\left[\frac{f}{d e - c f}\right]}\right] + \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & 12 f \operatorname{ArcCoth}[c+d x] \operatorname{PolyLog}\left[2, e^{\operatorname{ArcCoth}[c+d x]+\operatorname{ArcTanh}\left[\frac{f}{d e-c f}\right]}\right] + \\
 & 6 f \operatorname{ArcCoth}[c+d x] \operatorname{PolyLog}\left[2, e^{2\left(\operatorname{ArcCoth}[c+d x]+\operatorname{ArcTanh}\left[\frac{f}{d e-c f}\right]\right)}\right] - 6 f \operatorname{ArcCoth}[c+d x] \\
 & \operatorname{PolyLog}\left[2, \frac{e^{2 \operatorname{ArcCoth}[c+d x]}(d e+f-c f)}{d e-(1+c) f}\right] - 12 f \operatorname{PolyLog}\left[3, -e^{\operatorname{ArcCoth}[c+d x]+\operatorname{ArcTanh}\left[\frac{f}{d e-c f}\right]}\right] - \\
 & 12 f \operatorname{PolyLog}\left[3, e^{\operatorname{ArcCoth}[c+d x]+\operatorname{ArcTanh}\left[\frac{f}{d e-c f}\right]}\right] - 3 f \operatorname{PolyLog}\left[3, e^{2\left(\operatorname{ArcCoth}[c+d x]+\operatorname{ArcTanh}\left[\frac{f}{d e-c f}\right]\right)}\right] + \\
 & \left. 3 f \operatorname{PolyLog}\left[3, \frac{e^{2 \operatorname{ArcCoth}[c+d x]}(d e+f-c f)}{d e-(1+c) f}\right]\right)
 \end{aligned}$$

**Problem 113: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a+b \operatorname{ArcCoth}[c+d x])^2}{(e+f x)^2} dx$$

Optimal (type 4, 480 leaves, 24 steps):

$$\begin{aligned}
 & -\frac{(a+b \operatorname{ArcCoth}[c+d x])^2}{f(e+f x)} + \frac{b^2 d \operatorname{ArcCoth}[c+d x] \operatorname{Log}\left[\frac{2}{1-c-d x}\right]}{f(d e+f-c f)} - \frac{a b d \operatorname{Log}[1-c-d x]}{f(d e+f-c f)} - \\
 & \frac{b^2 d \operatorname{ArcCoth}[c+d x] \operatorname{Log}\left[\frac{2}{1+c+d x}\right]}{f(d e-f-c f)} + \frac{2 b^2 d \operatorname{ArcCoth}[c+d x] \operatorname{Log}\left[\frac{2}{1+c+d x}\right]}{(d e+f-c f)(d e-(1+c) f)} + \frac{a b d \operatorname{Log}[1+c+d x]}{f(d e-f-c f)} + \\
 & \frac{2 a b d \operatorname{Log}[e+f x]}{f^2-(d e-c f)^2} - \frac{2 b^2 d \operatorname{ArcCoth}[c+d x] \operatorname{Log}\left[\frac{2 d(e+f x)}{(d e+f-c f)(1+c+d x)}\right]}{(d e+f-c f)(d e-(1+c) f)} + \frac{b^2 d \operatorname{PolyLog}\left[2, -\frac{1+c+d x}{1-c-d x}\right]}{2 f(d e+f-c f)} + \\
 & \frac{b^2 d \operatorname{PolyLog}\left[2, 1-\frac{2}{1+c+d x}\right]}{2 f(d e-f-c f)} - \frac{b^2 d \operatorname{PolyLog}\left[2, 1-\frac{2}{1+c+d x}\right]}{(d e+f-c f)(d e-(1+c) f)} + \frac{b^2 d \operatorname{PolyLog}\left[2, 1-\frac{2 d(e+f x)}{(d e+f-c f)(1+c+d x)}\right]}{(d e+f-c f)(d e-(1+c) f)}
 \end{aligned}$$

Result (type 4, 806 leaves):

$$\begin{aligned}
 & -\frac{a^2}{f(e+f x)} + \frac{1}{d(e+f x)^2} 2 a b \left(1-(c+d x)^2\right) \left(\frac{f}{\sqrt{1-\frac{1}{(c+d x)^2}}} + \frac{d e-c f}{(c+d x) \sqrt{1-\frac{1}{(c+d x)^2}}}\right)^2 \\
 & \left(\frac{(-d e+c f) \operatorname{ArcCoth}[c+d x]}{f(-d e-f+c f)(-d e+f+c f)} - \operatorname{ArcCoth}[c+d x]\right) / \left(f(c+d x) \sqrt{1-\frac{1}{(c+d x)^2}}\right)
 \end{aligned}$$

$$\left( \left( -\frac{f}{\sqrt{1-\frac{1}{(c+dx)^2}}} - \frac{de}{(c+dx)\sqrt{1-\frac{1}{(c+dx)^2}}} + \frac{cf}{(c+dx)\sqrt{1-\frac{1}{(c+dx)^2}}} \right) + \right.$$

$$\left. \frac{\text{Log}\left[-\frac{f}{\sqrt{1-\frac{1}{(c+dx)^2}}} - \frac{de}{(c+dx)\sqrt{1-\frac{1}{(c+dx)^2}}} + \frac{cf}{(c+dx)\sqrt{1-\frac{1}{(c+dx)^2}}}\right]}{d^2 e^2 - 2cde f - f^2 + c^2 f^2} \right) +$$

$$\frac{1}{df(e+fx)^2} b^2 (1-(c+dx)^2) \left( \frac{f}{\sqrt{1-\frac{1}{(c+dx)^2}}} + \frac{de-cf}{(c+dx)\sqrt{1-\frac{1}{(c+dx)^2}}} \right)^2$$

$$\left( \frac{e^{\text{ArcTanh}\left[\frac{f}{-de+cf}\right]} \text{ArcCoth}[c+dx]^2}{(-de+cf)\sqrt{1-\frac{f^2}{(de-cf)^2}}} + \frac{\text{ArcCoth}[c+dx]^2}{(c+dx)\sqrt{1-\frac{1}{(c+dx)^2}}} \left( \frac{f}{\sqrt{1-\frac{1}{(c+dx)^2}}} + \frac{de-cf}{(c+dx)\sqrt{1-\frac{1}{(c+dx)^2}}} \right) \right) +$$

$$\frac{1}{d^2 e^2 - 2cde f + (-1+c^2) f^2} f \left( i \pi \text{ArcCoth}[c+dx] + 2 \text{ArcCoth}[c+dx] \text{ArcTanh}\left[\frac{f}{de-cf}\right] - \right.$$

$$i \pi \text{Log}\left[1 + e^{2 \text{ArcCoth}[c+dx]}\right] + 2 \text{ArcCoth}[c+dx] \text{Log}\left[1 - e^{-2(\text{ArcCoth}[c+dx] + \text{ArcTanh}\left[\frac{f}{de-cf}\right])}\right] -$$

$$2 \text{ArcTanh}\left[\frac{f}{-de+cf}\right] \text{Log}\left[1 - e^{-2(\text{ArcCoth}[c+dx] + \text{ArcTanh}\left[\frac{f}{de-cf}\right])}\right] + i \pi \text{Log}\left[\frac{1}{\sqrt{1-\frac{1}{(c+dx)^2}}}\right] +$$

$$2 \text{ArcTanh}\left[\frac{f}{-de+cf}\right] \text{Log}\left[i \text{Sinh}\left[\text{ArcCoth}[c+dx] + \text{ArcTanh}\left[\frac{f}{de-cf}\right]\right]\right] -$$

$$\left. \text{PolyLog}\left[2, e^{-2(\text{ArcCoth}[c+dx] + \text{ArcTanh}\left[\frac{f}{de-cf}\right])}\right] \right)$$

Problem 114: Result unnecessarily involves complex numbers and more than



twice size of optimal antiderivative.

$$\int (e + f x)^2 (a + b \operatorname{ArcCoth}[c + d x])^3 dx$$

Optimal (type 4, 546 leaves, 21 steps):

$$\begin{aligned} & \frac{a b^2 f^2 x}{d^2} + \frac{b^3 f^2 (c + d x) \operatorname{ArcCoth}[c + d x]}{d^3} - \\ & \frac{b f^2 (a + b \operatorname{ArcCoth}[c + d x])^2}{2 d^3} + \frac{3 b f (d e - c f) (a + b \operatorname{ArcCoth}[c + d x])^2}{d^3} + \\ & \frac{3 b f (d e - c f) (c + d x) (a + b \operatorname{ArcCoth}[c + d x])^2}{d^3} + \frac{b f^2 (c + d x)^2 (a + b \operatorname{ArcCoth}[c + d x])^2}{2 d^3} - \\ & \frac{(d e - c f) (d^2 e^2 - 2 c d e f + (3 + c^2) f^2) (a + b \operatorname{ArcCoth}[c + d x])^3}{3 d^3 f} + \\ & \frac{(3 d^2 e^2 - 6 c d e f + (1 + 3 c^2) f^2) (a + b \operatorname{ArcCoth}[c + d x])^3}{3 d^3} + \\ & \frac{(e + f x)^3 (a + b \operatorname{ArcCoth}[c + d x])^3}{3 f} - \frac{6 b^2 f (d e - c f) (a + b \operatorname{ArcCoth}[c + d x]) \operatorname{Log}\left[\frac{2}{1 - c - d x}\right]}{d^3} - \\ & \frac{1}{d^3} b (3 d^2 e^2 - 6 c d e f + (1 + 3 c^2) f^2) (a + b \operatorname{ArcCoth}[c + d x])^2 \operatorname{Log}\left[\frac{2}{1 - c - d x}\right] + \\ & \frac{b^3 f^2 \operatorname{Log}\left[1 - (c + d x)^2\right]}{2 d^3} - \frac{3 b^3 f (d e - c f) \operatorname{PolyLog}\left[2, -\frac{1 + c + d x}{1 - c - d x}\right]}{d^3} - \frac{1}{d^3} \\ & b^2 (3 d^2 e^2 - 6 c d e f + (1 + 3 c^2) f^2) (a + b \operatorname{ArcCoth}[c + d x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - c - d x}\right] + \\ & \frac{b^3 (3 d^2 e^2 - 6 c d e f + (1 + 3 c^2) f^2) \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 - c - d x}\right]}{2 d^3} \end{aligned}$$

Result (type 4, 2594 leaves):

$$\begin{aligned} & \frac{a^2 (a d^2 e^2 + 3 b d e f - 2 b c f^2) x}{d^2} + \frac{a^2 f (2 a d e + b f) x^2}{2 d} + \\ & \frac{1}{3} a^3 f^2 x^3 + a^2 b x (3 e^2 + 3 e f x + f^2 x^2) \operatorname{ArcCoth}[c + d x] + \\ & \frac{1}{2 d^3} (3 a^2 b d^2 e^2 - 3 a^2 b c d^2 e^2 + 3 a^2 b d e f - 6 a^2 b c d e f + \\ & 3 a^2 b c^2 d e f + a^2 b f^2 - 3 a^2 b c f^2 + 3 a^2 b c^2 f^2 - a^2 b c^3 f^2) \operatorname{Log}[1 - c - d x] + \\ & \frac{1}{2 d^3} (3 a^2 b d^2 e^2 + 3 a^2 b c d^2 e^2 - 3 a^2 b d e f - 6 a^2 b c d e f - 3 a^2 b c^2 d e f + a^2 b f^2 + \\ & 3 a^2 b c f^2 + 3 a^2 b c^2 f^2 + a^2 b c^3 f^2) \operatorname{Log}[1 + c + d x] + (3 a b^2 e^2 (1 - (c + d x)^2) \\ & (\operatorname{ArcCoth}[c + d x] (\operatorname{ArcCoth}[c + d x] - (c + d x) \operatorname{ArcCoth}[c + d x] + 2 \operatorname{Log}[1 - e^{-2 \operatorname{ArcCoth}[c + d x]}]) - \\ & \operatorname{PolyLog}[2, e^{-2 \operatorname{ArcCoth}[c + d x]}])) / \left( d (c + d x)^2 \left( 1 - \frac{1}{(c + d x)^2} \right) \right) - \end{aligned}$$

$$\begin{aligned}
& \left( 3 a b^2 e f \left( 1 - (c + d x)^2 \right) \left( 2 c \operatorname{ArcCoth}[c + d x]^2 + (c + d x)^2 \left( 1 - \frac{1}{(c + d x)^2} \right) \operatorname{ArcCoth}[c + d x]^2 - \right. \right. \\
& 2 (c + d x) \operatorname{ArcCoth}[c + d x] (-1 + c \operatorname{ArcCoth}[c + d x]) + \\
& \left. 4 c \operatorname{ArcCoth}[c + d x] \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcCoth}[c + d x]}\right] - 2 \operatorname{Log}\left[\frac{1}{(c + d x) \sqrt{1 - \frac{1}{(c + d x)^2}}}\right] - \right. \\
& \left. \left. 2 c \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcCoth}[c + d x]}\right] \right) \right) / \left( d^2 (c + d x)^2 \left( 1 - \frac{1}{(c + d x)^2} \right) \right) + \\
& \left( b^3 e^2 \left( 1 - (c + d x)^2 \right) \left( \frac{i \pi^3}{8} - \operatorname{ArcCoth}[c + d x]^3 - (c + d x) \operatorname{ArcCoth}[c + d x]^3 + \right. \right. \\
& 3 \operatorname{ArcCoth}[c + d x]^2 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcCoth}[c + d x]}\right] + 3 \operatorname{ArcCoth}[c + d x] \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcCoth}[c + d x]}\right] - \\
& \left. \left. \frac{3}{2} \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcCoth}[c + d x]}\right] \right) \right) / \left( d (c + d x)^2 \left( 1 - \frac{1}{(c + d x)^2} \right) \right) - \\
& \frac{1}{4 d^2 (c + d x)^2 \left( 1 - \frac{1}{(c + d x)^2} \right)} b^3 e f \left( 1 - (c + d x)^2 \right) \left( i c \pi^3 - 12 \operatorname{ArcCoth}[c + d x]^2 + \right. \\
& 12 (c + d x) \operatorname{ArcCoth}[c + d x]^2 - 8 c \operatorname{ArcCoth}[c + d x]^3 - 8 c (c + d x) \operatorname{ArcCoth}[c + d x]^3 + \\
& 4 (c + d x)^2 \left( 1 - \frac{1}{(c + d x)^2} \right) \operatorname{ArcCoth}[c + d x]^3 - 24 \operatorname{ArcCoth}[c + d x] \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcCoth}[c + d x]}\right] + \\
& 24 c \operatorname{ArcCoth}[c + d x]^2 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcCoth}[c + d x]}\right] + 12 \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcCoth}[c + d x]}\right] + \\
& \left. 24 c \operatorname{ArcCoth}[c + d x] \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcCoth}[c + d x]}\right] - 12 c \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcCoth}[c + d x]}\right] \right) - \\
& \frac{1}{4 d^3} a b^2 f^2 (c + d x) \sqrt{1 - \frac{1}{(c + d x)^2}} \left( 1 - (c + d x)^2 \right) \left( \frac{4 \operatorname{ArcCoth}[c + d x]}{(c + d x) \sqrt{1 - \frac{1}{(c + d x)^2}}} + \right. \\
& \frac{3 \operatorname{ArcCoth}[c + d x]^2}{(c + d x) \sqrt{1 - \frac{1}{(c + d x)^2}}} - \frac{12 c \operatorname{ArcCoth}[c + d x]^2}{(c + d x) \sqrt{1 - \frac{1}{(c + d x)^2}}} + \frac{9 c^2 \operatorname{ArcCoth}[c + d x]^2}{(c + d x) \sqrt{1 - \frac{1}{(c + d x)^2}}} + \\
& \left. \frac{1}{\sqrt{1 - \frac{1}{(c + d x)^2}}} (-1 + 6 c \operatorname{ArcCoth}[c + d x] + 3 \operatorname{ArcCoth}[c + d x]^2 - 3 c^2 \operatorname{ArcCoth}[c + d x]^2) + \right. \\
& \left. \operatorname{Cosh}\left[3 \operatorname{ArcCoth}[c + d x]\right] - 6 c \operatorname{ArcCoth}[c + d x] \operatorname{Cosh}\left[3 \operatorname{ArcCoth}[c + d x]\right] + \right. \\
& \left. \operatorname{ArcCoth}[c + d x]^2 \operatorname{Cosh}\left[3 \operatorname{ArcCoth}[c + d x]\right] + 3 c^2 \operatorname{ArcCoth}[c + d x]^2 \operatorname{Cosh}\left[3 \operatorname{ArcCoth}[c + d x]\right] + \right.
\end{aligned}$$

$$\begin{aligned}
 & \frac{6 \operatorname{ArcCoth}[c + d x] \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcCoth}[c + d x]}\right]}{(c + d x) \sqrt{1 - \frac{1}{(c + d x)^2}}} + \frac{18 c^2 \operatorname{ArcCoth}[c + d x] \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcCoth}[c + d x]}\right]}{(c + d x) \sqrt{1 - \frac{1}{(c + d x)^2}}} - \\
 & \frac{18 c \operatorname{Log}\left[\frac{1}{(c + d x) \sqrt{1 - \frac{1}{(c + d x)^2}}}\right]}{(c + d x) \sqrt{1 - \frac{1}{(c + d x)^2}}} + \frac{4 (1 + 3 c^2) \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcCoth}[c + d x]}\right]}{(c + d x)^3 \left(1 - \frac{1}{(c + d x)^2}\right)^{3/2}} - \\
 & \left. \begin{aligned}
 & \operatorname{ArcCoth}[c + d x]^2 \operatorname{Sinh}\left[3 \operatorname{ArcCoth}[c + d x]\right] - 3 c^2 \operatorname{ArcCoth}[c + d x]^2 \operatorname{Sinh}\left[3 \operatorname{ArcCoth}[c + d x]\right] - \\
 & 2 \operatorname{ArcCoth}[c + d x] \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcCoth}[c + d x]}\right] \operatorname{Sinh}\left[3 \operatorname{ArcCoth}[c + d x]\right] - \\
 & 6 c^2 \operatorname{ArcCoth}[c + d x] \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcCoth}[c + d x]}\right] \operatorname{Sinh}\left[3 \operatorname{ArcCoth}[c + d x]\right] + \\
 & 6 c \operatorname{Log}\left[\frac{1}{(c + d x) \sqrt{1 - \frac{1}{(c + d x)^2}}}\right] \operatorname{Sinh}\left[3 \operatorname{ArcCoth}[c + d x]\right] \left. \right\} + \\
 & \frac{1}{d^3 (c + d x)^2 \left(1 - \frac{1}{(c + d x)^2}\right)} b^3 f^2 \left(1 - (c + d x)^2\right) \left(3 c \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcCoth}[c + d x]}\right] + \right. \\
 & \left. \frac{1}{96} (c + d x)^3 \left(1 - \frac{1}{(c + d x)^2}\right)^{3/2} \left(-\frac{3 i \pi^3}{(c + d x) \sqrt{1 - \frac{1}{(c + d x)^2}}} - \frac{9 i c^2 \pi^3}{(c + d x) \sqrt{1 - \frac{1}{(c + d x)^2}}}\right) + \right. \\
 & \frac{24 \operatorname{ArcCoth}[c + d x]}{\sqrt{1 - \frac{1}{(c + d x)^2}}} - \frac{72 c \operatorname{ArcCoth}[c + d x]^2}{\sqrt{1 - \frac{1}{(c + d x)^2}}} - \frac{48 \operatorname{ArcCoth}[c + d x]^2}{(c + d x) \sqrt{1 - \frac{1}{(c + d x)^2}}} + \\
 & \frac{216 c \operatorname{ArcCoth}[c + d x]^2}{(c + d x) \sqrt{1 - \frac{1}{(c + d x)^2}}} - \frac{24 \operatorname{ArcCoth}[c + d x]^3}{\sqrt{1 - \frac{1}{(c + d x)^2}}} + \frac{24 c^2 \operatorname{ArcCoth}[c + d x]^3}{\sqrt{1 - \frac{1}{(c + d x)^2}}} + \\
 & \frac{24 \operatorname{ArcCoth}[c + d x]^3}{(c + d x) \sqrt{1 - \frac{1}{(c + d x)^2}}} + \frac{96 c \operatorname{ArcCoth}[c + d x]^3}{(c + d x) \sqrt{1 - \frac{1}{(c + d x)^2}}} + \frac{72 c^2 \operatorname{ArcCoth}[c + d x]^3}{(c + d x) \sqrt{1 - \frac{1}{(c + d x)^2}}} - 24 \operatorname{ArcCoth}[ \\
 & c + d x] \operatorname{Cosh}\left[3 \operatorname{ArcCoth}[c + d x]\right] + 72 c \operatorname{ArcCoth}[c + d x]^2 \operatorname{Cosh}\left[3 \operatorname{ArcCoth}[c + d x]\right] - \\
 & 8 \operatorname{ArcCoth}[c + d x]^3 \operatorname{Cosh}\left[3 \operatorname{ArcCoth}[c + d x]\right] - 24 c^2 \operatorname{ArcCoth}[c + d x]^3 \\
 & \operatorname{Cosh}\left[3 \operatorname{ArcCoth}[c + d x]\right] + \frac{432 c \operatorname{ArcCoth}[c + d x] \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcCoth}[c + d x]}\right]}{(c + d x) \sqrt{1 - \frac{1}{(c + d x)^2}}} -
 \end{aligned}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{72 \operatorname{ArcCoth}[c + d x]^2 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcCoth}[c + d x]}\right]}{(c + d x) \sqrt{1 - \frac{1}{(c + d x)^2}}} - \\
 & \frac{216 c^2 \operatorname{ArcCoth}[c + d x]^2 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcCoth}[c + d x]}\right]}{(c + d x) \sqrt{1 - \frac{1}{(c + d x)^2}}} - \frac{72 \operatorname{Log}\left[\frac{1}{(c + d x) \sqrt{1 - \frac{1}{(c + d x)^2}}}\right]}{(c + d x) \sqrt{1 - \frac{1}{(c + d x)^2}}} + \\
 & \frac{96 (1 + 3 c^2) \operatorname{ArcCoth}[c + d x] \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcCoth}[c + d x]}\right]}{(c + d x)^3 \left(1 - \frac{1}{(c + d x)^2}\right)^{3/2}} - \\
 & \frac{48 (1 + 3 c^2) \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcCoth}[c + d x]}\right]}{(c + d x)^3 \left(1 - \frac{1}{(c + d x)^2}\right)^{3/2}} + \frac{3 \pi^3 \operatorname{Sinh}\left[3 \operatorname{ArcCoth}[c + d x]\right]}{3 \pi^3 \operatorname{Sinh}\left[3 \operatorname{ArcCoth}[c + d x]\right]} + \\
 & - 72 c \operatorname{ArcCoth}[c + d x]^2 \operatorname{Sinh}\left[3 \operatorname{ArcCoth}[c + d x]\right] - \\
 & - 8 \operatorname{ArcCoth}[c + d x]^3 \operatorname{Sinh}\left[3 \operatorname{ArcCoth}[c + d x]\right] - \\
 & - 24 c^2 \operatorname{ArcCoth}[c + d x]^3 \operatorname{Sinh}\left[3 \operatorname{ArcCoth}[c + d x]\right] - \\
 & + 144 c \operatorname{ArcCoth}[c + d x] \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcCoth}[c + d x]}\right] \operatorname{Sinh}\left[3 \operatorname{ArcCoth}[c + d x]\right] + \\
 & + 24 \operatorname{ArcCoth}[c + d x]^2 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcCoth}[c + d x]}\right] \operatorname{Sinh}\left[3 \operatorname{ArcCoth}[c + d x]\right] + \\
 & + 72 c^2 \operatorname{ArcCoth}[c + d x]^2 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcCoth}[c + d x]}\right] \operatorname{Sinh}\left[3 \operatorname{ArcCoth}[c + d x]\right] + \\
 & \left. \left. \left. 24 \operatorname{Log}\left[\frac{1}{(c + d x) \sqrt{1 - \frac{1}{(c + d x)^2}}}\right] \operatorname{Sinh}\left[3 \operatorname{ArcCoth}[c + d x]\right]\right)\right)\right)
 \end{aligned}$$

**Problem 115: Result unnecessarily involves imaginary or complex numbers.**

$$\int (e + f x) (a + b \operatorname{ArcCoth}[c + d x])^3 dx$$

Optimal (type 4, 326 leaves, 15 steps):

$$\begin{aligned}
 & \frac{3 b f (a + b \operatorname{ArcCoth}[c + d x])^2}{2 d^2} + \frac{3 b f (c + d x) (a + b \operatorname{ArcCoth}[c + d x])^2}{2 d^2} + \\
 & \frac{(d e - c f) (a + b \operatorname{ArcCoth}[c + d x])^3}{d^2} - \frac{(d^2 e^2 - 2 c d e f + (1 + c^2) f^2) (a + b \operatorname{ArcCoth}[c + d x])^3}{2 d^2 f} + \\
 & \frac{(e + f x)^2 (a + b \operatorname{ArcCoth}[c + d x])^3}{2 f} - \frac{3 b^2 f (a + b \operatorname{ArcCoth}[c + d x]) \operatorname{Log}\left[\frac{2}{1 - c - d x}\right]}{d^2} - \\
 & \frac{3 b (d e - c f) (a + b \operatorname{ArcCoth}[c + d x])^2 \operatorname{Log}\left[\frac{2}{1 - c - d x}\right]}{d^2} - \frac{3 b^3 f \operatorname{PolyLog}\left[2, -\frac{1 + c + d x}{1 - c - d x}\right]}{2 d^2} - \\
 & \frac{3 b^2 (d e - c f) (a + b \operatorname{ArcCoth}[c + d x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - c - d x}\right]}{d^2} + \\
 & \frac{3 b^3 (d e - c f) \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 - c - d x}\right]}{2 d^2}
 \end{aligned}$$

Result (type 4, 600 leaves):

$$\begin{aligned}
& \frac{1}{4 d^2} \left( 2 a^2 (2 a d e + 3 b f - 2 a c f) (c + d x) + 2 a^3 f (c + d x)^2 - \right. \\
& 6 a^2 b (c + d x) (c f - d (2 e + f x)) \operatorname{ArcCoth}[c + d x] + 3 a^2 b (2 d e + f - 2 c f) \operatorname{Log}[1 - c - d x] + \\
& 3 a^2 b (2 d e - (1 + 2 c) f) \operatorname{Log}[1 + c + d x] + 12 a b^2 f \left( (c + d x) \operatorname{ArcCoth}[c + d x] + \right. \\
& \left. \frac{1}{2} (-1 + (c + d x)^2) \operatorname{ArcCoth}[c + d x] - \operatorname{Log}\left[ \frac{1}{(c + d x) \sqrt{1 - \frac{1}{(c + d x)^2}}} \right] \right) + \\
& 12 a b^2 d e (\operatorname{ArcCoth}[c + d x] ((-1 + c + d x) \operatorname{ArcCoth}[c + d x] - 2 \operatorname{Log}[1 - e^{-2 \operatorname{ArcCoth}[c + d x]}]) + \\
& \operatorname{PolyLog}[2, e^{-2 \operatorname{ArcCoth}[c + d x]}]) - \\
& 12 a b^2 c f (\operatorname{ArcCoth}[c + d x] ((-1 + c + d x) \operatorname{ArcCoth}[c + d x] - 2 \operatorname{Log}[1 - e^{-2 \operatorname{ArcCoth}[c + d x]}]) + \\
& \operatorname{PolyLog}[2, e^{-2 \operatorname{ArcCoth}[c + d x]}]) + \\
& 2 b^3 f (\operatorname{ArcCoth}[c + d x] (3 (-1 + c + d x) \operatorname{ArcCoth}[c + d x] + (-1 + c^2 + 2 c d x + d^2 x^2) \\
& \operatorname{ArcCoth}[c + d x]^2 - 6 \operatorname{Log}[1 - e^{-2 \operatorname{ArcCoth}[c + d x]}]) + 3 \operatorname{PolyLog}[2, e^{-2 \operatorname{ArcCoth}[c + d x]}]) + \\
& 4 b^3 d e \left( -\frac{i \pi^3}{8} + \operatorname{ArcCoth}[c + d x]^3 + (c + d x) \operatorname{ArcCoth}[c + d x]^3 - \right. \\
& 3 \operatorname{ArcCoth}[c + d x]^2 \operatorname{Log}[1 - e^{2 \operatorname{ArcCoth}[c + d x]}] - \\
& 3 \operatorname{ArcCoth}[c + d x] \operatorname{PolyLog}[2, e^{2 \operatorname{ArcCoth}[c + d x]}] + \frac{3}{2} \operatorname{PolyLog}[3, e^{2 \operatorname{ArcCoth}[c + d x]}] \left. \right) - \\
& 4 b^3 c f \left( -\frac{i \pi^3}{8} + \operatorname{ArcCoth}[c + d x]^3 + (c + d x) \operatorname{ArcCoth}[c + d x]^3 - \right. \\
& 3 \operatorname{ArcCoth}[c + d x]^2 \operatorname{Log}[1 - e^{2 \operatorname{ArcCoth}[c + d x]}] - \\
& \left. 3 \operatorname{ArcCoth}[c + d x] \operatorname{PolyLog}[2, e^{2 \operatorname{ArcCoth}[c + d x]}] + \frac{3}{2} \operatorname{PolyLog}[3, e^{2 \operatorname{ArcCoth}[c + d x]}] \right) \left. \right)
\end{aligned}$$

**Problem 116: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a + b \operatorname{ArcCoth}[c + d x])^3 dx$$

Optimal (type 4, 132 leaves, 6 steps):

$$\frac{(a + b \operatorname{ArcCoth}[c + d x])^3}{d} + \frac{(c + d x) (a + b \operatorname{ArcCoth}[c + d x])^3}{d} - \frac{3 b (a + b \operatorname{ArcCoth}[c + d x])^2 \operatorname{Log}\left[\frac{2}{1 - c - d x}\right]}{d} - \frac{3 b^2 (a + b \operatorname{ArcCoth}[c + d x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - c - d x}\right]}{d} + \frac{3 b^3 \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 - c - d x}\right]}{2 d}$$

Result (type 4, 208 leaves):

$$\frac{1}{2 d} \left( 2 a^3 (c + d x) + 6 a^2 b (c + d x) \operatorname{ArcCoth}[c + d x] + 3 a^2 b \operatorname{Log}\left[1 - (c + d x)^2\right] + 6 a b^2 (\operatorname{ArcCoth}[c + d x] ((-1 + c + d x) \operatorname{ArcCoth}[c + d x] - 2 \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcCoth}[c + d x]}\right]) + \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcCoth}[c + d x]}\right]) + 2 b^3 \left(-\frac{i \pi^3}{8} + \operatorname{ArcCoth}[c + d x]^3 + (c + d x) \operatorname{ArcCoth}[c + d x]^3 - 3 \operatorname{ArcCoth}[c + d x]^2 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcCoth}[c + d x]}\right] - 3 \operatorname{ArcCoth}[c + d x] \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcCoth}[c + d x]}\right] + \frac{3}{2} \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcCoth}[c + d x]}\right]\right) \right)$$

### Problem 117: Unable to integrate problem.

$$\int \frac{(a + b \operatorname{ArcCoth}[c + d x])^3}{e + f x} dx$$

Optimal (type 4, 308 leaves, 2 steps):

$$-\frac{(a + b \operatorname{ArcCoth}[c + d x])^3 \operatorname{Log}\left[\frac{2}{1 + c + d x}\right]}{f} + \frac{(a + b \operatorname{ArcCoth}[c + d x])^3 \operatorname{Log}\left[\frac{2 d (e + f x)}{(d e + f - c f) (1 + c + d x)}\right]}{f} - \frac{3 b (a + b \operatorname{ArcCoth}[c + d x])^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + c + d x}\right]}{2 f} - \frac{3 b (a + b \operatorname{ArcCoth}[c + d x])^2 \operatorname{PolyLog}\left[2, 1 - \frac{2 d (e + f x)}{(d e + f - c f) (1 + c + d x)}\right]}{2 f} + \frac{3 b^2 (a + b \operatorname{ArcCoth}[c + d x]) \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 + c + d x}\right]}{2 f} - \frac{3 b^2 (a + b \operatorname{ArcCoth}[c + d x]) \operatorname{PolyLog}\left[3, 1 - \frac{2 d (e + f x)}{(d e + f - c f) (1 + c + d x)}\right]}{2 f} + \frac{3 b^3 \operatorname{PolyLog}\left[4, 1 - \frac{2}{1 + c + d x}\right]}{4 f} - \frac{3 b^3 \operatorname{PolyLog}\left[4, 1 - \frac{2 d (e + f x)}{(d e + f - c f) (1 + c + d x)}\right]}{4 f}$$

Result (type 8, 22 leaves):

$$\int \frac{(a + b \operatorname{ArcCoth}[c + d x])^3}{e + f x} dx$$

### Problem 118: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b \operatorname{ArcCoth}[c + dx])^3}{(e + fx)^2} dx$$

Optimal (type 4, 1089 leaves, 33 steps):

$$\begin{aligned} & -\frac{(a + b \operatorname{ArcCoth}[c + dx])^3}{f(e + fx)} + \frac{3ab^2d \operatorname{ArcCoth}[c + dx] \operatorname{Log}\left[\frac{2}{1 - c - dx}\right]}{f(de + f - cf)} + \\ & \frac{3b^3d \operatorname{ArcCoth}[c + dx]^2 \operatorname{Log}\left[\frac{2}{1 - c - dx}\right]}{2f(de + f - cf)} - \frac{3a^2bd \operatorname{Log}[1 - c - dx]}{2f(de + f - cf)} - \\ & \frac{3ab^2d \operatorname{ArcCoth}[c + dx] \operatorname{Log}\left[\frac{2}{1 + c + dx}\right]}{f(de - f - cf)} + \frac{6ab^2d \operatorname{ArcCoth}[c + dx] \operatorname{Log}\left[\frac{2}{1 + c + dx}\right]}{(de + f - cf)(de - (1 + c)f)} - \\ & \frac{3b^3d \operatorname{ArcCoth}[c + dx]^2 \operatorname{Log}\left[\frac{2}{1 + c + dx}\right]}{2f(de - f - cf)} + \frac{3b^3d \operatorname{ArcCoth}[c + dx]^2 \operatorname{Log}\left[\frac{2}{1 + c + dx}\right]}{(de + f - cf)(de - (1 + c)f)} + \\ & \frac{3a^2bd \operatorname{Log}[1 + c + dx]}{2f(de - f - cf)} + \frac{3a^2bd \operatorname{Log}[e + fx]}{f^2 - (de - cf)^2} - \frac{6ab^2d \operatorname{ArcCoth}[c + dx] \operatorname{Log}\left[\frac{2d(e + fx)}{(de + f - cf)(1 + c + dx)}\right]}{(de + f - cf)(de - (1 + c)f)} - \\ & \frac{3b^3d \operatorname{ArcCoth}[c + dx]^2 \operatorname{Log}\left[\frac{2d(e + fx)}{(de + f - cf)(1 + c + dx)}\right]}{(de + f - cf)(de - (1 + c)f)} + \frac{3ab^2d \operatorname{PolyLog}\left[2, -\frac{1 + c + dx}{1 - c - dx}\right]}{2f(de + f - cf)} + \\ & \frac{3b^3d \operatorname{ArcCoth}[c + dx] \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - c - dx}\right]}{2f(de + f - cf)} + \frac{3ab^2d \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + c + dx}\right]}{2f(de - f - cf)} - \\ & \frac{3ab^2d \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + c + dx}\right]}{(de + f - cf)(de - (1 + c)f)} + \frac{3b^3d \operatorname{ArcCoth}[c + dx] \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + c + dx}\right]}{2f(de - f - cf)} - \\ & \frac{3b^3d \operatorname{ArcCoth}[c + dx] \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + c + dx}\right]}{(de + f - cf)(de - (1 + c)f)} + \frac{3ab^2d \operatorname{PolyLog}\left[2, 1 - \frac{2d(e + fx)}{(de + f - cf)(1 + c + dx)}\right]}{(de + f - cf)(de - (1 + c)f)} + \\ & \frac{3b^3d \operatorname{ArcCoth}[c + dx] \operatorname{PolyLog}\left[2, 1 - \frac{2d(e + fx)}{(de + f - cf)(1 + c + dx)}\right]}{(de + f - cf)(de - (1 + c)f)} - \\ & \frac{3b^3d \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 - c - dx}\right]}{4f(de + f - cf)} + \frac{3b^3d \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 + c + dx}\right]}{4f(de - f - cf)} - \\ & \frac{3b^3d \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 + c + dx}\right]}{2(de + f - cf)(de - (1 + c)f)} + \frac{3b^3d \operatorname{PolyLog}\left[3, 1 - \frac{2d(e + fx)}{(de + f - cf)(1 + c + dx)}\right]}{2(de + f - cf)(de - (1 + c)f)} \end{aligned}$$

Result (type 4, 1816 leaves):

$$-\frac{a^3}{f(e + fx)} - \frac{3a^2b \operatorname{ArcCoth}[c + dx]}{f(e + fx)} + \frac{3a^2bd \operatorname{Log}[1 - c - dx]}{2f(-de - f + cf)} -$$



$$\begin{aligned}
 & \frac{3 a^2 b d \operatorname{Log}[1+c+d x]}{2 f(-d e+f+c f)} - \frac{3 a^2 b d \operatorname{Log}[e+f x]}{d^2 e^2-2 c d e f-f^2+c^2 f^2} + \\
 & \frac{1}{d f(e+f x)^2} 3 a b^2\left(1-(c+d x)^2\right)\left(\frac{f}{\sqrt{1-\frac{1}{(c+d x)^2}}}+\frac{d e-c f}{(c+d x) \sqrt{1-\frac{1}{(c+d x)^2}}}\right)^2 \\
 & \left(\frac{e^{\operatorname{ArcTanh}\left[\frac{f}{-d e-c f}\right]} \operatorname{ArcCoth}[c+d x]^2}{(-d e+c f) \sqrt{1-\frac{f^2}{(d e-c f)^2}}}+\frac{\operatorname{ArcCoth}[c+d x]^2}{(c+d x) \sqrt{1-\frac{1}{(c+d x)^2}}}\left(\frac{f}{\sqrt{1-\frac{1}{(c+d x)^2}}}+\frac{d e-c f}{(c+d x) \sqrt{1-\frac{1}{(c+d x)^2}}}\right)\right)+ \\
 & \frac{1}{d^2 e^2-2 c d e f+(-1+c^2) f^2} f\left(i \pi \operatorname{ArcCoth}[c+d x]+2 \operatorname{ArcCoth}[c+d x] \operatorname{ArcTanh}\left[\frac{f}{d e-c f}\right]-\right. \\
 & i \pi \operatorname{Log}\left[1+e^{2 \operatorname{ArcCoth}[c+d x]}\right]+2 \operatorname{ArcCoth}[c+d x] \operatorname{Log}\left[1-e^{-2\left(\operatorname{ArcCoth}[c+d x]+\operatorname{ArcTanh}\left[\frac{f}{d e-c f}\right]\right)}\right]- \\
 & 2 \operatorname{ArcTanh}\left[\frac{f}{-d e+c f}\right] \operatorname{Log}\left[1-e^{-2\left(\operatorname{ArcCoth}[c+d x]+\operatorname{ArcTanh}\left[\frac{f}{d e-c f}\right]\right)}\right]+i \pi \operatorname{Log}\left[\frac{1}{\sqrt{1-\frac{1}{(c+d x)^2}}}\right]+ \\
 & 2 \operatorname{ArcTanh}\left[\frac{f}{-d e+c f}\right] \operatorname{Log}\left[i \operatorname{Sinh}\left[\operatorname{ArcCoth}[c+d x]+\operatorname{ArcTanh}\left[\frac{f}{d e-c f}\right]\right]\right]- \\
 & \left.\left.\operatorname{PolyLog}\left[2, e^{-2\left(\operatorname{ArcCoth}[c+d x]+\operatorname{ArcTanh}\left[\frac{f}{d e-c f}\right]\right)}\right]\right)\right]+ \\
 & \frac{1}{d(e+f x)^2} b^3\left(1-(c+d x)^2\right)\left(\frac{f}{\sqrt{1-\frac{1}{(c+d x)^2}}}+\frac{d e-c f}{(c+d x) \sqrt{1-\frac{1}{(c+d x)^2}}}\right)^2 \\
 & \left(-\left(\operatorname{ArcCoth}[c+d x]^3\right) / \left(f(c+d x) \sqrt{1-\frac{1}{(c+d x)^2}}\right)\right)
 \end{aligned}$$

$$\begin{aligned}
& \left( -\frac{f}{\sqrt{1 - \frac{1}{(c+dx)^2}}} - \frac{de}{(c+dx)\sqrt{1 - \frac{1}{(c+dx)^2}}} + \frac{cf}{(c+dx)\sqrt{1 - \frac{1}{(c+dx)^2}}} \right) + \\
& \frac{1}{2f(de+f-cf)(de-(1+c)f)} \left( 2de \operatorname{ArcCoth}[c+dx]^3 - 6f \operatorname{ArcCoth}[c+dx]^3 - \right. \\
& 2cf \operatorname{ArcCoth}[c+dx]^3 - 4de e^{-\operatorname{ArcTanh}\left[\frac{f}{de-cf}\right]} \sqrt{\frac{d^2e^2 - 2cdef + (-1+c^2)f^2}{(de-cf)^2}} \\
& \operatorname{ArcCoth}[c+dx]^3 + 4c e^{-\operatorname{ArcTanh}\left[\frac{f}{de-cf}\right]} f \sqrt{\frac{d^2e^2 - 2cdef + (-1+c^2)f^2}{(de-cf)^2}} \operatorname{ArcCoth}[c+dx]^3 + \\
& 6if\pi \operatorname{ArcCoth}[c+dx] \operatorname{Log}[2] - f \operatorname{ArcCoth}[c+dx]^2 \operatorname{Log}[64] - 6if\pi \operatorname{ArcCoth}[c+dx] \operatorname{Log}\left[ \right. \\
& \left. e^{-\operatorname{ArcCoth}[c+dx]} + e^{\operatorname{ArcCoth}[c+dx]} \right] + 6f \operatorname{ArcCoth}[c+dx]^2 \operatorname{Log}\left[ 1 - e^{\operatorname{ArcCoth}[c+dx] + \operatorname{ArcTanh}\left[\frac{f}{de-cf}\right]} \right] + \\
& 6f \operatorname{ArcCoth}[c+dx]^2 \operatorname{Log}\left[ 1 + e^{\operatorname{ArcCoth}[c+dx] + \operatorname{ArcTanh}\left[\frac{f}{de-cf}\right]} \right] + 6f \operatorname{ArcCoth}[c+dx]^2 \\
& \operatorname{Log}\left[ 1 - e^{2\left(\operatorname{ArcCoth}[c+dx] + \operatorname{ArcTanh}\left[\frac{f}{de-cf}\right]\right)} \right] + 12f \operatorname{ArcCoth}[c+dx] \operatorname{ArcTanh}\left[\frac{f}{de-cf}\right] \\
& \operatorname{Log}\left[ \frac{1}{2} e^{-\operatorname{ArcCoth}[c+dx] - \operatorname{ArcTanh}\left[\frac{f}{de-cf}\right]} \left( -1 + e^{2\left(\operatorname{ArcCoth}[c+dx] + \operatorname{ArcTanh}\left[\frac{f}{de-cf}\right]\right)} \right) \right] + \\
& 6f \operatorname{ArcCoth}[c+dx]^2 \operatorname{Log}\left[ -e^{-\operatorname{ArcCoth}[c+dx]} \left( de \left( -1 + e^{2\operatorname{ArcCoth}[c+dx]} \right) + \left( 1 + c + \right. \right. \right. \\
& \left. \left. \left. e^{2\operatorname{ArcCoth}[c+dx]} - c e^{2\operatorname{ArcCoth}[c+dx]} \right) f \right) \right] - 6f \operatorname{ArcCoth}[c+dx]^2 \operatorname{Log}\left[ \frac{1}{de - (1+c)f} \right. \\
& \left. \left( -de \left( -1 + e^{2\operatorname{ArcCoth}[c+dx]} \right) + \left( -1 - e^{2\operatorname{ArcCoth}[c+dx]} + c \left( -1 + e^{2\operatorname{ArcCoth}[c+dx]} \right) \right) f \right) \right] + \\
& 6if\pi \operatorname{ArcCoth}[c+dx] \operatorname{Log}\left[ \frac{1}{\sqrt{1 - \frac{1}{(c+dx)^2}}} \right] - 6f \operatorname{ArcCoth}[c+dx]^2 \\
& \operatorname{Log}\left[ -\frac{f}{\sqrt{1 - \frac{1}{(c+dx)^2}}} - \frac{de}{(c+dx)\sqrt{1 - \frac{1}{(c+dx)^2}}} + \frac{cf}{(c+dx)\sqrt{1 - \frac{1}{(c+dx)^2}}} \right] - 12f \\
& \operatorname{ArcCoth}[c+dx] \operatorname{ArcTanh}\left[\frac{f}{de-cf}\right] \operatorname{Log}\left[ i \operatorname{Sinh}\left[ \operatorname{ArcCoth}[c+dx] + \operatorname{ArcTanh}\left[\frac{f}{de-cf}\right] \right] \right] + \\
& 12f \operatorname{ArcCoth}[c+dx] \operatorname{PolyLog}\left[ 2, -e^{\operatorname{ArcCoth}[c+dx] + \operatorname{ArcTanh}\left[\frac{f}{de-cf}\right]} \right] + \\
& 12f \operatorname{ArcCoth}[c+dx] \operatorname{PolyLog}\left[ 2, e^{\operatorname{ArcCoth}[c+dx] + \operatorname{ArcTanh}\left[\frac{f}{de-cf}\right]} \right] + \\
& 6f \operatorname{ArcCoth}[c+dx] \operatorname{PolyLog}\left[ 2, e^{2\left(\operatorname{ArcCoth}[c+dx] + \operatorname{ArcTanh}\left[\frac{f}{de-cf}\right]\right)} \right] - 6f \operatorname{ArcCoth}[c+dx] \\
& \operatorname{PolyLog}\left[ 2, \frac{e^{2\operatorname{ArcCoth}[c+dx]}(de+f-cf)}{de - (1+c)f} \right] - 12f \operatorname{PolyLog}\left[ 3, -e^{\operatorname{ArcCoth}[c+dx] + \operatorname{ArcTanh}\left[\frac{f}{de-cf}\right]} \right] - \\
& 12f \operatorname{PolyLog}\left[ 3, e^{\operatorname{ArcCoth}[c+dx] + \operatorname{ArcTanh}\left[\frac{f}{de-cf}\right]} \right] - 3f \operatorname{PolyLog}\left[ 3, e^{2\left(\operatorname{ArcCoth}[c+dx] + \operatorname{ArcTanh}\left[\frac{f}{de-cf}\right]\right)} \right] +
\end{aligned}$$

$$\left. \left. \left. 3 f \operatorname{PolyLog}\left[3, \frac{e^{2 \operatorname{ArcCoth}[c+d x]} (d e+f-c f)}{d e-(1+c) f}\right]\right]\right)\right)$$

**Problem 119: Unable to integrate problem.**

$$\int (e+f x)^m (a+b \operatorname{ArcCoth}[c+d x]) dx$$

Optimal (type 5, 162 leaves, 6 steps):

$$\frac{(e+f x)^{1+m} (a+b \operatorname{ArcCoth}[c+d x])}{f(1+m)} + \frac{b d (e+f x)^{2+m} \operatorname{Hypergeometric2F1}\left[1, 2+m, 3+m, \frac{d(e+f x)}{d e-f-c f}\right]}{2 f (d e-(1+c) f) (1+m) (2+m)}$$

$$\frac{b d (e+f x)^{2+m} \operatorname{Hypergeometric2F1}\left[1, 2+m, 3+m, \frac{d(e+f x)}{d e+f-c f}\right]}{2 f (d e+f-c f) (1+m) (2+m)}$$

Result (type 8, 20 leaves):

$$\int (e+f x)^m (a+b \operatorname{ArcCoth}[c+d x]) dx$$

**Problem 123: Unable to integrate problem.**

$$\int \frac{\left(a+b \operatorname{ArcCoth}\left[\frac{\sqrt{1-c x}}{\sqrt{1+c x}}\right]\right)^3}{1-c^2 x^2} dx$$

Optimal (type 4, 460 leaves, 9 steps):

$$\begin{aligned}
 & \frac{2 \left( a + b \operatorname{ArcCoth} \left[ \frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right] \right)^3 \operatorname{ArcCoth} \left[ 1 - \frac{2}{1 + \frac{\sqrt{1-cx}}{\sqrt{1+cx}}} \right]}{c} \\
 & + \frac{3 b \left( a + b \operatorname{ArcCoth} \left[ \frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right] \right)^2 \operatorname{PolyLog} \left[ 2, 1 - \frac{2}{1 + \frac{\sqrt{1-cx}}{\sqrt{1+cx}}} \right]}{2 c} \\
 & - \frac{3 b \left( a + b \operatorname{ArcCoth} \left[ \frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right] \right)^2 \operatorname{PolyLog} \left[ 2, 1 - \frac{2 \sqrt{1-cx}}{\sqrt{1+cx} \left( 1 + \frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right)} \right]}{2 c} \\
 & + \frac{3 b^2 \left( a + b \operatorname{ArcCoth} \left[ \frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right] \right) \operatorname{PolyLog} \left[ 3, 1 - \frac{2}{1 + \frac{\sqrt{1-cx}}{\sqrt{1+cx}}} \right]}{2 c} \\
 & - \frac{3 b^2 \left( a + b \operatorname{ArcCoth} \left[ \frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right] \right) \operatorname{PolyLog} \left[ 3, 1 - \frac{2 \sqrt{1-cx}}{\sqrt{1+cx} \left( 1 + \frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right)} \right]}{2 c} \\
 & + \frac{3 b^3 \operatorname{PolyLog} \left[ 4, 1 - \frac{2}{1 + \frac{\sqrt{1-cx}}{\sqrt{1+cx}}} \right]}{4 c} + \frac{3 b^3 \operatorname{PolyLog} \left[ 4, 1 - \frac{2 \sqrt{1-cx}}{\sqrt{1+cx} \left( 1 + \frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right)} \right]}{4 c}
 \end{aligned}$$

Result (type 8, 42 leaves):

$$\int \frac{\left( a + b \operatorname{ArcCoth} \left[ \frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right] \right)^3}{1 - c^2 x^2} dx$$

Problem 124: Unable to integrate problem.

$$\int \frac{\left( a + b \operatorname{ArcCoth} \left[ \frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right] \right)^2}{1 - c^2 x^2} dx$$

Optimal (type 4, 302 leaves, 7 steps):

$$\frac{2 \left( a + b \operatorname{ArcCoth} \left[ \frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right] \right)^2 \operatorname{ArcCoth} \left[ 1 - \frac{2}{1 + \frac{\sqrt{1-cx}}{\sqrt{1+cx}}} \right]}{c} - \frac{b \left( a + b \operatorname{ArcCoth} \left[ \frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right] \right) \operatorname{PolyLog} \left[ 2, 1 - \frac{2}{1 + \frac{\sqrt{1-cx}}{\sqrt{1+cx}}} \right]}{c} + \frac{b \left( a + b \operatorname{ArcCoth} \left[ \frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right] \right) \operatorname{PolyLog} \left[ 2, 1 - \frac{2\sqrt{1-cx}}{\sqrt{1+cx} \left( 1 + \frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right)} \right]}{c} - \frac{b^2 \operatorname{PolyLog} \left[ 3, 1 - \frac{2}{1 + \frac{\sqrt{1-cx}}{\sqrt{1+cx}}} \right]}{2c} + \frac{b^2 \operatorname{PolyLog} \left[ 3, 1 - \frac{2\sqrt{1-cx}}{\sqrt{1+cx} \left( 1 + \frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right)} \right]}{2c}$$

Result (type 8, 42 leaves):

$$\int \frac{\left( a + b \operatorname{ArcCoth} \left[ \frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right] \right)^2}{1 - c^2 x^2} dx$$

**Problem 139: Result more than twice size of optimal antiderivative.**

$$\int x \operatorname{ArcCoth} [\operatorname{Tanh} [a + b x]]^2 dx$$

Optimal (type 3, 34 leaves, 3 steps):

$$\frac{x \operatorname{ArcCoth} [\operatorname{Tanh} [a + b x]]^3}{3b} - \frac{\operatorname{ArcCoth} [\operatorname{Tanh} [a + b x]]^4}{12b^2}$$

Result (type 3, 74 leaves):

$$\frac{1}{12b^2} (a + b x) \left( - (3a - b x) (a + b x)^2 + 4 (2a^2 + a b x - b^2 x^2) \operatorname{ArcCoth} [\operatorname{Tanh} [a + b x]] - 6 (a - b x) \operatorname{ArcCoth} [\operatorname{Tanh} [a + b x]]^2 \right)$$

**Problem 150: Result more than twice size of optimal antiderivative.**

$$\int x \operatorname{ArcCoth} [\operatorname{Tanh} [a + b x]]^3 dx$$

Optimal (type 3, 34 leaves, 3 steps):

$$\frac{x \operatorname{ArcCoth} [\operatorname{Tanh} [a + b x]]^4}{4b} - \frac{\operatorname{ArcCoth} [\operatorname{Tanh} [a + b x]]^5}{20b^2}$$

Result (type 3, 99 leaves):

$$\frac{1}{20b^2} (a+bx) \left( (4a-bx)(a+bx)^3 - 5(3a-bx)(a+bx)^2 \operatorname{ArcCoth}[\operatorname{Tanh}[a+bx]] + 10(2a^2+abx-b^2x^2) \operatorname{ArcCoth}[\operatorname{Tanh}[a+bx]]^2 - 10(a-bx) \operatorname{ArcCoth}[\operatorname{Tanh}[a+bx]]^3 \right)$$

### Problem 205: Result more than twice size of optimal antiderivative.

$$\int \operatorname{ArcCoth}[c+d \operatorname{Tanh}[a+bx]] dx$$

Optimal (type 4, 150 leaves, 7 steps):

$$x \operatorname{ArcCoth}[c+d \operatorname{Tanh}[a+bx]] + \frac{1}{2} x \operatorname{Log}\left[1 + \frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right] - \frac{1}{2} x \operatorname{Log}\left[1 + \frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right] + \frac{\operatorname{PolyLog}\left[2, -\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right]}{4b} - \frac{\operatorname{PolyLog}\left[2, -\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right]}{4b}$$

Result (type 4, 366 leaves):

$$x \operatorname{ArcCoth}[c+d \operatorname{Tanh}[a+bx]] + \frac{1}{2b} \left( (a+bx) \operatorname{Log}\left[1 - \frac{\sqrt{-1+c+d} e^{a+bx}}{\sqrt{1-c+d}}\right] + (a+bx) \operatorname{Log}\left[1 + \frac{\sqrt{-1+c+d} e^{a+bx}}{\sqrt{1-c+d}}\right] - (a+bx) \operatorname{Log}\left[1 - \frac{\sqrt{1+c+d} e^{a+bx}}{\sqrt{-1-c+d}}\right] - (a+bx) \operatorname{Log}\left[1 + \frac{\sqrt{1+c+d} e^{a+bx}}{\sqrt{-1-c+d}}\right] + a \operatorname{Log}\left[1+c-d+e^{2(a+bx)}+c e^{2(a+bx)}+d e^{2(a+bx)}\right] - a \operatorname{Log}\left[1+d+e^{2(a+bx)}-d e^{2(a+bx)}-c(1+e^{2(a+bx)})\right] + \operatorname{PolyLog}\left[2, -\frac{\sqrt{-1+c+d} e^{a+bx}}{\sqrt{1-c+d}}\right] + \operatorname{PolyLog}\left[2, \frac{\sqrt{-1+c+d} e^{a+bx}}{\sqrt{1-c+d}}\right] - \operatorname{PolyLog}\left[2, -\frac{\sqrt{1+c+d} e^{a+bx}}{\sqrt{-1-c+d}}\right] - \operatorname{PolyLog}\left[2, \frac{\sqrt{1+c+d} e^{a+bx}}{\sqrt{-1-c+d}}\right] \right)$$

### Problem 210: Result more than twice size of optimal antiderivative.

$$\int \operatorname{ArcCoth}[1+d+d \operatorname{Tanh}[a+bx]] dx$$

Optimal (type 4, 69 leaves, 5 steps):

$$\frac{bx^2}{2} + x \operatorname{ArcCoth}[1+d+d \operatorname{Tanh}[a+bx]] - \frac{1}{2} x \operatorname{Log}\left[1 + (1+d) e^{2a+2bx}\right] - \frac{\operatorname{PolyLog}\left[2, -(1+d) e^{2a+2bx}\right]}{4b}$$

Result (type 4, 168 leaves):

$$\begin{aligned}
 & x \operatorname{ArcCoth}[1 + d + d \operatorname{Tanh}[a + b x]] - \frac{1}{2b} \\
 & \left( b x \left( -b x - \operatorname{Log}\left[ e^{-a-bx} + (1+d) e^{a+bx} \right] + \operatorname{Log}\left[ 1 - e^{bx} \sqrt{-(1+d) e^{2a}} \right] + \right. \right. \\
 & \quad \left. \left. \operatorname{Log}\left[ 1 + e^{bx} \sqrt{-(1+d) e^{2a}} \right] + \operatorname{Log}\left[ (2+d) \operatorname{Cosh}[a + b x] + d \operatorname{Sinh}[a + b x] \right] \right) + \right. \\
 & \quad \left. \operatorname{PolyLog}\left[ 2, -e^{bx} \sqrt{-(1+d) e^{2a}} \right] + \operatorname{PolyLog}\left[ 2, e^{bx} \sqrt{-(1+d) e^{2a}} \right] \right)
 \end{aligned}$$

### Problem 215: Result more than twice size of optimal antiderivative.

$$\int \operatorname{ArcCoth}[1 - d - d \operatorname{Tanh}[a + b x]] dx$$

Optimal (type 4, 76 leaves, 5 steps):

$$\frac{b x^2}{2} + x \operatorname{ArcCoth}[1 - d - d \operatorname{Tanh}[a + b x]] - \frac{1}{2} x \operatorname{Log}\left[ 1 + (1-d) e^{2a+2bx} \right] - \frac{\operatorname{PolyLog}\left[ 2, -(1-d) e^{2a+2bx} \right]}{4b}$$

Result (type 4, 171 leaves):

$$\begin{aligned}
 & x \operatorname{ArcCoth}[1 - d - d \operatorname{Tanh}[a + b x]] - \frac{1}{2b} \\
 & \left( b x \left( -b x - \operatorname{Log}\left[ e^{-a-bx} (-1 + (-1+d) e^{2(a+bx)}) \right] + \operatorname{Log}\left[ 1 - e^{bx} \sqrt{(-1+d) e^{2a}} \right] + \right. \right. \\
 & \quad \left. \left. \operatorname{Log}\left[ 1 + e^{bx} \sqrt{(-1+d) e^{2a}} \right] + \operatorname{Log}\left[ (-2+d) \operatorname{Cosh}[a + b x] + d \operatorname{Sinh}[a + b x] \right] \right) + \right. \\
 & \quad \left. \operatorname{PolyLog}\left[ 2, -e^{bx} \sqrt{(-1+d) e^{2a}} \right] + \operatorname{PolyLog}\left[ 2, e^{bx} \sqrt{(-1+d) e^{2a}} \right] \right)
 \end{aligned}$$

### Problem 219: Result more than twice size of optimal antiderivative.

$$\int \operatorname{ArcCoth}[c + d \operatorname{Coth}[a + b x]] dx$$

Optimal (type 4, 150 leaves, 7 steps):

$$\begin{aligned}
 & x \operatorname{ArcCoth}[c + d \operatorname{Coth}[a + b x]] + \frac{1}{2} x \operatorname{Log}\left[ 1 - \frac{(1-c-d) e^{2a+2bx}}{1-c+d} \right] - \\
 & \frac{1}{2} x \operatorname{Log}\left[ 1 - \frac{(1+c+d) e^{2a+2bx}}{1+c-d} \right] + \frac{\operatorname{PolyLog}\left[ 2, \frac{(1-c-d) e^{2a+2bx}}{1-c+d} \right]}{4b} - \frac{\operatorname{PolyLog}\left[ 2, \frac{(1+c+d) e^{2a+2bx}}{1+c-d} \right]}{4b}
 \end{aligned}$$

Result (type 4, 369 leaves):

$$\begin{aligned}
& x \operatorname{ArcCoth}[c + d \operatorname{Coth}[a + b x]] - \\
& \frac{1}{2b} \left( - (a + b x) \operatorname{Log}\left[1 - \frac{\sqrt{-1 + c + d} e^{a+bx}}{\sqrt{-1 + c - d}}\right] - (a + b x) \operatorname{Log}\left[1 + \frac{\sqrt{-1 + c + d} e^{a+bx}}{\sqrt{-1 + c - d}}\right] + \right. \\
& \quad (a + b x) \operatorname{Log}\left[1 - \frac{\sqrt{1 + c + d} e^{a+bx}}{\sqrt{1 + c - d}}\right] + (a + b x) \operatorname{Log}\left[1 + \frac{\sqrt{1 + c + d} e^{a+bx}}{\sqrt{1 + c - d}}\right] + \\
& \quad a \operatorname{Log}\left[1 + d - e^{2(a+bx)} + d e^{2(a+bx)} + c(-1 + e^{2(a+bx)})\right] - \\
& \quad a \operatorname{Log}\left[1 + c - e^{2(a+bx)} - c e^{2(a+bx)} - d(1 + e^{2(a+bx)})\right] - \\
& \quad \operatorname{PolyLog}\left[2, -\frac{\sqrt{-1 + c + d} e^{a+bx}}{\sqrt{-1 + c - d}}\right] - \operatorname{PolyLog}\left[2, \frac{\sqrt{-1 + c + d} e^{a+bx}}{\sqrt{-1 + c - d}}\right] + \\
& \quad \left. \operatorname{PolyLog}\left[2, -\frac{\sqrt{1 + c + d} e^{a+bx}}{\sqrt{1 + c - d}}\right] + \operatorname{PolyLog}\left[2, \frac{\sqrt{1 + c + d} e^{a+bx}}{\sqrt{1 + c - d}}\right] \right)
\end{aligned}$$

### Problem 224: Result more than twice size of optimal antiderivative.

$$\int \operatorname{ArcCoth}[1 + d + d \operatorname{Coth}[a + b x]] dx$$

Optimal (type 4, 69 leaves, 5 steps):

$$\frac{bx^2}{2} + x \operatorname{ArcCoth}[1 + d + d \operatorname{Coth}[a + b x]] - \frac{1}{2} x \operatorname{Log}[1 - (1 + d) e^{2a+2bx}] - \frac{\operatorname{PolyLog}[2, (1 + d) e^{2a+2bx}]}{4b}$$

Result (type 4, 168 leaves):

$$\begin{aligned}
& x \operatorname{ArcCoth}[1 + d + d \operatorname{Coth}[a + b x]] - \frac{1}{2b} \\
& \left( b x \left( -b x - \operatorname{Log}[e^{-a-bx}(-1 + (1 + d) e^{2(a+bx)})] \right) + \operatorname{Log}\left[1 - e^{bx} \sqrt{(1 + d) e^{2a}}\right] + \right. \\
& \quad \left. \operatorname{Log}\left[1 + e^{bx} \sqrt{(1 + d) e^{2a}}\right] + \operatorname{Log}\left[d \operatorname{Cosh}[a + b x] + (2 + d) \operatorname{Sinh}[a + b x]\right] \right) + \\
& \quad \operatorname{PolyLog}\left[2, -e^{bx} \sqrt{(1 + d) e^{2a}}\right] + \operatorname{PolyLog}\left[2, e^{bx} \sqrt{(1 + d) e^{2a}}\right]
\end{aligned}$$

### Problem 229: Result more than twice size of optimal antiderivative.

$$\int \operatorname{ArcCoth}[1 - d - d \operatorname{Coth}[a + b x]] dx$$

Optimal (type 4, 76 leaves, 5 steps):

$$\frac{bx^2}{2} + x \operatorname{ArcCoth}[1 - d - d \operatorname{Coth}[a + b x]] - \frac{1}{2} x \operatorname{Log}[1 - (1 - d) e^{2a+2bx}] - \frac{\operatorname{PolyLog}[2, (1 - d) e^{2a+2bx}]}{4b}$$

Result (type 4, 175 leaves):



$$\begin{aligned}
 & x \operatorname{ArcCoth}[1 - d - d \operatorname{Coth}[a + b x]] - \frac{1}{2b} \\
 & \left( b x \left( -b x - \operatorname{Log}\left[ e^{-a-bx} \left( 1 + (-1+d) e^{2(a+bx)} \right) \right] + \operatorname{Log}\left[ 1 - e^{bx} \sqrt{-(-1+d) e^{2a}} \right] + \right. \right. \\
 & \quad \left. \left. \operatorname{Log}\left[ 1 + e^{bx} \sqrt{-(-1+d) e^{2a}} \right] + \operatorname{Log}\left[ d \operatorname{Cosh}[a + b x] + (-2+d) \operatorname{Sinh}[a + b x] \right] \right) + \right. \\
 & \quad \left. \operatorname{PolyLog}\left[ 2, -e^{bx} \sqrt{-(-1+d) e^{2a}} \right] + \operatorname{PolyLog}\left[ 2, e^{bx} \sqrt{-(-1+d) e^{2a}} \right] \right)
 \end{aligned}$$

### Problem 231: Result more than twice size of optimal antiderivative.

$$\int (e + f x)^3 \operatorname{ArcCoth}[\operatorname{Tan}[a + b x]] dx$$

Optimal (type 4, 302 leaves, 12 steps):

$$\begin{aligned}
 & \frac{(e + f x)^4 \operatorname{ArcCoth}[\operatorname{Tan}[a + b x]]}{4 f} + \frac{i (e + f x)^4 \operatorname{ArcTan}\left[ e^{2i(a+bx)} \right]}{4 f} - \\
 & \frac{i (e + f x)^3 \operatorname{PolyLog}\left[ 2, -i e^{2i(a+bx)} \right]}{4 b} + \frac{i (e + f x)^3 \operatorname{PolyLog}\left[ 2, i e^{2i(a+bx)} \right]}{4 b} + \\
 & \frac{3 f (e + f x)^2 \operatorname{PolyLog}\left[ 3, -i e^{2i(a+bx)} \right]}{8 b^2} - \frac{3 f (e + f x)^2 \operatorname{PolyLog}\left[ 3, i e^{2i(a+bx)} \right]}{8 b^2} + \\
 & \frac{3 i f^2 (e + f x) \operatorname{PolyLog}\left[ 4, -i e^{2i(a+bx)} \right]}{8 b^3} - \frac{3 i f^2 (e + f x) \operatorname{PolyLog}\left[ 4, i e^{2i(a+bx)} \right]}{8 b^3} - \\
 & \frac{3 f^3 \operatorname{PolyLog}\left[ 5, -i e^{2i(a+bx)} \right]}{16 b^4} + \frac{3 f^3 \operatorname{PolyLog}\left[ 5, i e^{2i(a+bx)} \right]}{16 b^4}
 \end{aligned}$$

Result (type 4, 654 leaves):

$$\begin{aligned}
 & \frac{1}{4} x \left( 4 e^3 + 6 e^2 f x + 4 e f^2 x^2 + f^3 x^3 \right) \operatorname{ArcCoth}[\operatorname{Tan}[a + b x]] + \\
 & \frac{1}{16 b^4} \left( -8 b^4 e^3 x \operatorname{Log}\left[ 1 - i e^{2i(a+bx)} \right] - 12 b^4 e^2 f x^2 \operatorname{Log}\left[ 1 - i e^{2i(a+bx)} \right] - \right. \\
 & \quad 8 b^4 e f^2 x^3 \operatorname{Log}\left[ 1 - i e^{2i(a+bx)} \right] - 2 b^4 f^3 x^4 \operatorname{Log}\left[ 1 - i e^{2i(a+bx)} \right] + 8 b^4 e^3 x \operatorname{Log}\left[ 1 + i e^{2i(a+bx)} \right] + \\
 & \quad 12 b^4 e^2 f x^2 \operatorname{Log}\left[ 1 + i e^{2i(a+bx)} \right] + 8 b^4 e f^2 x^3 \operatorname{Log}\left[ 1 + i e^{2i(a+bx)} \right] + \\
 & \quad 2 b^4 f^3 x^4 \operatorname{Log}\left[ 1 + i e^{2i(a+bx)} \right] - 4 i b^3 (e + f x)^3 \operatorname{PolyLog}\left[ 2, -i e^{2i(a+bx)} \right] + \\
 & \quad 4 i b^3 (e + f x)^3 \operatorname{PolyLog}\left[ 2, i e^{2i(a+bx)} \right] + 6 b^2 e^2 f \operatorname{PolyLog}\left[ 3, -i e^{2i(a+bx)} \right] + \\
 & \quad 12 b^2 e f^2 x \operatorname{PolyLog}\left[ 3, -i e^{2i(a+bx)} \right] + 6 b^2 f^3 x^2 \operatorname{PolyLog}\left[ 3, -i e^{2i(a+bx)} \right] - \\
 & \quad 6 b^2 e^2 f \operatorname{PolyLog}\left[ 3, i e^{2i(a+bx)} \right] - 12 b^2 e f^2 x \operatorname{PolyLog}\left[ 3, i e^{2i(a+bx)} \right] - \\
 & \quad 6 b^2 f^3 x^2 \operatorname{PolyLog}\left[ 3, i e^{2i(a+bx)} \right] + 6 i b e f^2 \operatorname{PolyLog}\left[ 4, -i e^{2i(a+bx)} \right] + \\
 & \quad 6 i b f^3 x \operatorname{PolyLog}\left[ 4, -i e^{2i(a+bx)} \right] - 6 i b e f^2 \operatorname{PolyLog}\left[ 4, i e^{2i(a+bx)} \right] - \\
 & \quad \left. 6 i b f^3 x \operatorname{PolyLog}\left[ 4, i e^{2i(a+bx)} \right] - 3 f^3 \operatorname{PolyLog}\left[ 5, -i e^{2i(a+bx)} \right] + 3 f^3 \operatorname{PolyLog}\left[ 5, i e^{2i(a+bx)} \right] \right)
 \end{aligned}$$

### Problem 238: Result more than twice size of optimal antiderivative.

$$\int \operatorname{ArcCoth}[c + d \operatorname{Tan}[a + b x]] dx$$

Optimal (type 4, 194 leaves, 7 steps):

$$\begin{aligned}
& x \operatorname{ArcCoth}[c + d \operatorname{Tan}[a + b x]] + \\
& \frac{1}{2} x \operatorname{Log}\left[1 + \frac{(1 - c + i d) e^{2ia + 2ibx}}{1 - c - i d}\right] - \frac{1}{2} x \operatorname{Log}\left[1 + \frac{(1 + c - i d) e^{2ia + 2ibx}}{1 + c + i d}\right] - \\
& \frac{i \operatorname{PolyLog}\left[2, -\frac{(1 - c + i d) e^{2ia + 2ibx}}{1 - c - i d}\right]}{4 b} + \frac{i \operatorname{PolyLog}\left[2, -\frac{(1 + c - i d) e^{2ia + 2ibx}}{1 + c + i d}\right]}{4 b}
\end{aligned}$$

Result (type 4, 4654 leaves):

$$\begin{aligned}
& x \operatorname{ArcCoth}[c + d \operatorname{Tan}[a + b x]] + \\
& \left( d \left[ -a \operatorname{Log}\left[-\operatorname{Sec}\left[\frac{1}{2}(a + b x)\right]^2 ((-1 + c) \operatorname{Cos}[a + b x] + d \operatorname{Sin}[a + b x])\right]\right] + \right. \\
& a \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{2}(a + b x)\right]^2 (\operatorname{Cos}[a + b x] + c \operatorname{Cos}[a + b x] + d \operatorname{Sin}[a + b x])\right]\right] + \\
& (a + b x) \operatorname{Log}\left[\frac{-d + \sqrt{1 - 2c + c^2 + d^2}}{-1 + c} + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right] + \\
& i \operatorname{Log}\left[\frac{(-1 + c) \left(1 + i \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right)}{-1 + c + i d - i \sqrt{1 - 2c + c^2 + d^2}}\right] \operatorname{Log}\left[\frac{-d + \sqrt{1 - 2c + c^2 + d^2}}{-1 + c} + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right] - \\
& i \operatorname{Log}\left[-\frac{(-1 + c) \left(i + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right)}{i - i c - d + \sqrt{1 - 2c + c^2 + d^2}}\right] \operatorname{Log}\left[\frac{-d + \sqrt{1 - 2c + c^2 + d^2}}{-1 + c} + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right] + \\
& (a + b x) \operatorname{Log}\left[\frac{d + \sqrt{1 - 2c + c^2 + d^2}}{1 - c} + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right] + \\
& i \operatorname{Log}\left[\frac{(-1 + c) \left(-i + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right)}{i - i c + d + \sqrt{1 - 2c + c^2 + d^2}}\right] \operatorname{Log}\left[\frac{d + \sqrt{1 - 2c + c^2 + d^2}}{1 - c} + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right] - \\
& i \operatorname{Log}\left[\frac{(-1 + c) \left(i + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right)}{-i + i c + d + \sqrt{1 - 2c + c^2 + d^2}}\right] \operatorname{Log}\left[\frac{d + \sqrt{1 - 2c + c^2 + d^2}}{1 - c} + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right] - \\
& (a + b x) \operatorname{Log}\left[-\frac{d + \sqrt{1 + 2c + c^2 + d^2}}{1 + c} + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right] - \\
& i \operatorname{Log}\left[\frac{(1 + c) \left(-i + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right)}{-i - i c + d + \sqrt{1 + 2c + c^2 + d^2}}\right] \operatorname{Log}\left[-\frac{d + \sqrt{1 + 2c + c^2 + d^2}}{1 + c} + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right] + \\
& i \operatorname{Log}\left[\frac{(1 + c) \left(i + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right)}{i + i c + d + \sqrt{1 + 2c + c^2 + d^2}}\right] \operatorname{Log}\left[-\frac{d + \sqrt{1 + 2c + c^2 + d^2}}{1 + c} + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right] - \\
& (a + b x) \operatorname{Log}\left[\frac{-d + \sqrt{1 + 2c + c^2 + d^2} + (1 + c) \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]}{1 + c}\right] + \\
& i \operatorname{Log}\left[\frac{(1 + c) \left(1 - i \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right)}{1 + c - i d + i \sqrt{1 + 2c + c^2 + d^2}}\right] \\
& \operatorname{Log}\left[\frac{-d + \sqrt{1 + 2c + c^2 + d^2} + (1 + c) \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]}{1 + c}\right] - i \operatorname{Log}\left[
\end{aligned}$$

$$\begin{aligned}
 & \left. \frac{(1+c) \left(1 + i \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)}{1+c+i d-i \sqrt{1+2c+c^2+d^2}} \right] \operatorname{Log}\left[\frac{-d+\sqrt{1+2c+c^2+d^2}+(1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{1+c}\right] + \\
 & i \operatorname{PolyLog}\left[2, \frac{d+\sqrt{1-2c+c^2+d^2}-(-1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{i-i c+d+\sqrt{1-2c+c^2+d^2}}\right] - \\
 & i \operatorname{PolyLog}\left[2, \frac{d+\sqrt{1-2c+c^2+d^2}-(-1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{-i+i c+d+\sqrt{1-2c+c^2+d^2}}\right] - \\
 & i \operatorname{PolyLog}\left[2, \frac{-d+\sqrt{1-2c+c^2+d^2}+(-1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{i-i c-d+\sqrt{1-2c+c^2+d^2}}\right] + \\
 & i \operatorname{PolyLog}\left[2, \frac{-d+\sqrt{1-2c+c^2+d^2}+(-1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{-i+i c-d+\sqrt{1-2c+c^2+d^2}}\right] - \\
 & i \operatorname{PolyLog}\left[2, \frac{d+\sqrt{1+2c+c^2+d^2}-(1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{-i-i c+d+\sqrt{1+2c+c^2+d^2}}\right] + \\
 & i \operatorname{PolyLog}\left[2, \frac{d+\sqrt{1+2c+c^2+d^2}-(1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{i+i c+d+\sqrt{1+2c+c^2+d^2}}\right] + \\
 & i \operatorname{PolyLog}\left[2, \frac{-d+\sqrt{1+2c+c^2+d^2}+(1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{-i-i c-d+\sqrt{1+2c+c^2+d^2}}\right] - \\
 & i \operatorname{PolyLog}\left[2, \frac{-d+\sqrt{1+2c+c^2+d^2}+(1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{i+i c-d+\sqrt{1+2c+c^2+d^2}}\right] \Big) \\
 & \left. \left(-\left(\frac{2a}{b(-1+c^2+d^2-\operatorname{Cos}[2(a+bx)]+c^2 \operatorname{Cos}[2(a+bx)]-d^2 \operatorname{Cos}[2(a+bx)]+2cd \operatorname{Sin}[2(a+bx)])}\right)\right)+\left(\frac{2(a+bx)}{b(-1+c^2+d^2-\operatorname{Cos}[2(a+bx)]+c^2 \operatorname{Cos}[2(a+bx)]-d^2 \operatorname{Cos}[2(a+bx)]+2cd \operatorname{Sin}[2(a+bx)])}\right)\right)\right) \Big) / \\
 & \left( \operatorname{Log}\left[\frac{-d+\sqrt{1-2c+c^2+d^2}}{-1+c}+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right]+\operatorname{Log}\left[\frac{d+\sqrt{1-2c+c^2+d^2}}{1-c}+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right]- \right. \\
 & \operatorname{Log}\left[-\frac{d+\sqrt{1+2c+c^2+d^2}}{1+c}+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right]- \\
 & \operatorname{Log}\left[\frac{-d+\sqrt{1+2c+c^2+d^2}+(1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{1+c}\right]+ \\
 & \left. \frac{\operatorname{Log}\left[\frac{-d+\sqrt{1+2c+c^2+d^2}+(1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{1+c}\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(1-i \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} \right)
 \end{aligned}$$

$$\begin{aligned}
 & \frac{\text{Log}\left[\frac{-d+\sqrt{1-2c+c^2+d^2}}{-1+c} + \text{Tan}\left[\frac{1}{2}(a+bx)\right]\right] \text{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(1+i\text{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} + \\
 & \frac{\text{Log}\left[\frac{-d+\sqrt{1+2c+c^2+d^2}+(1+c)\text{Tan}\left[\frac{1}{2}(a+bx)\right]}{1+c}\right] \text{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(1+i\text{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} + \\
 & \frac{i\text{Log}\left[\frac{d+\sqrt{1-2c+c^2+d^2}}{1-c} + \text{Tan}\left[\frac{1}{2}(a+bx)\right]\right] \text{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-i+\text{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} - \\
 & \frac{i\text{Log}\left[-\frac{d+\sqrt{1+2c+c^2+d^2}}{1+c} + \text{Tan}\left[\frac{1}{2}(a+bx)\right]\right] \text{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-i+\text{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} - \\
 & \frac{i\text{Log}\left[\frac{-d+\sqrt{1-2c+c^2+d^2}}{-1+c} + \text{Tan}\left[\frac{1}{2}(a+bx)\right]\right] \text{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(i+\text{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} - \\
 & \frac{i\text{Log}\left[\frac{d+\sqrt{1-2c+c^2+d^2}}{1-c} + \text{Tan}\left[\frac{1}{2}(a+bx)\right]\right] \text{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(i+\text{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} + \\
 & \frac{i\text{Log}\left[-\frac{d+\sqrt{1+2c+c^2+d^2}}{1+c} + \text{Tan}\left[\frac{1}{2}(a+bx)\right]\right] \text{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(i+\text{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} + \\
 & \frac{(a+bx)\text{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(\frac{-d+\sqrt{1-2c+c^2+d^2}}{-1+c} + \text{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} + \\
 & \frac{i\text{Log}\left[\frac{(-1+c)\left(1+i\text{Tan}\left[\frac{1}{2}(a+bx)\right]\right)}{-1+c+i d-i\sqrt{1-2c+c^2+d^2}}\right] \text{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(\frac{-d+\sqrt{1-2c+c^2+d^2}}{-1+c} + \text{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} - \\
 & \frac{i\text{Log}\left[\frac{(-1+c)\left(i+\text{Tan}\left[\frac{1}{2}(a+bx)\right]\right)}{i-i c-d+\sqrt{1-2c+c^2+d^2}}\right] \text{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(\frac{-d+\sqrt{1-2c+c^2+d^2}}{-1+c} + \text{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} + \\
 & \frac{(a+bx)\text{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(\frac{d+\sqrt{1-2c+c^2+d^2}}{1-c} + \text{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} +
 \end{aligned}$$

$$\begin{aligned}
 & \frac{i \operatorname{Log} \left[ \frac{(-1+c) \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)}{i - i c + d + \sqrt{1-2c+c^2+d^2}} \right] \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2}{2 \left( \frac{d + \sqrt{1-2c+c^2+d^2}}{1-c} + \operatorname{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)} - \\
 & \frac{i \operatorname{Log} \left[ \frac{(-1+c) \left( i + \operatorname{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)}{-i + i c + d + \sqrt{1-2c+c^2+d^2}} \right] \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2}{2 \left( \frac{d + \sqrt{1-2c+c^2+d^2}}{1-c} + \operatorname{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)} - \\
 & \frac{(a+bx) \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2}{2 \left( -\frac{d + \sqrt{1+2c+c^2+d^2}}{1+c} + \operatorname{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)} - \\
 & \frac{i \operatorname{Log} \left[ \frac{(1+c) \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)}{-i - i c + d + \sqrt{1+2c+c^2+d^2}} \right] \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2}{2 \left( -\frac{d + \sqrt{1+2c+c^2+d^2}}{1+c} + \operatorname{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)} + \\
 & \frac{i \operatorname{Log} \left[ \frac{(1+c) \left( i + \operatorname{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)}{i + i c + d + \sqrt{1+2c+c^2+d^2}} \right] \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2}{2 \left( -\frac{d + \sqrt{1+2c+c^2+d^2}}{1+c} + \operatorname{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)} + \\
 & \left( i (-1+c) \operatorname{Log} \left[ 1 - \frac{d + \sqrt{1-2c+c^2+d^2} - (-1+c) \operatorname{Tan} \left[ \frac{1}{2} (a+bx) \right]}{i - i c + d + \sqrt{1-2c+c^2+d^2}} \right] \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \right) / \\
 & \left( 2 \left( d + \sqrt{1-2c+c^2+d^2} - (-1+c) \operatorname{Tan} \left[ \frac{1}{2} (a+bx) \right] \right) \right) - \\
 & \left( i (-1+c) \operatorname{Log} \left[ 1 - \frac{d + \sqrt{1-2c+c^2+d^2} - (-1+c) \operatorname{Tan} \left[ \frac{1}{2} (a+bx) \right]}{-i + i c + d + \sqrt{1-2c+c^2+d^2}} \right] \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \right) / \\
 & \left( 2 \left( d + \sqrt{1-2c+c^2+d^2} - (-1+c) \operatorname{Tan} \left[ \frac{1}{2} (a+bx) \right] \right) \right) + \\
 & \left( i (-1+c) \operatorname{Log} \left[ 1 - \frac{-d + \sqrt{1-2c+c^2+d^2} + (-1+c) \operatorname{Tan} \left[ \frac{1}{2} (a+bx) \right]}{i - i c - d + \sqrt{1-2c+c^2+d^2}} \right] \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \right) / \\
 & \left( 2 \left( -d + \sqrt{1-2c+c^2+d^2} + (-1+c) \operatorname{Tan} \left[ \frac{1}{2} (a+bx) \right] \right) \right) - \\
 & \left( i (-1+c) \operatorname{Log} \left[ 1 - \frac{-d + \sqrt{1-2c+c^2+d^2} + (-1+c) \operatorname{Tan} \left[ \frac{1}{2} (a+bx) \right]}{-i + i c - d + \sqrt{1-2c+c^2+d^2}} \right] \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \right) / \\
 & \left( 2 \left( -d + \sqrt{1-2c+c^2+d^2} + (-1+c) \operatorname{Tan} \left[ \frac{1}{2} (a+bx) \right] \right) \right) -
 \end{aligned}$$

$$\begin{aligned}
 & \frac{i(1+c) \operatorname{Log}\left[1 - \frac{d + \sqrt{1+2c+c^2+d^2} - (1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{-i-i c+d+\sqrt{1+2c+c^2+d^2}}\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(d + \sqrt{1+2c+c^2+d^2} - (1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} + \\
 & \frac{i(1+c) \operatorname{Log}\left[1 - \frac{d + \sqrt{1+2c+c^2+d^2} - (1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{i+i c+d+\sqrt{1+2c+c^2+d^2}}\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(d + \sqrt{1+2c+c^2+d^2} - (1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} - \\
 & \frac{(1+c)(a+bx) \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-d + \sqrt{1+2c+c^2+d^2} + (1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} + \\
 & \frac{i(1+c) \operatorname{Log}\left[\frac{(1+c)(1-i \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right])}{1+c-i d+i \sqrt{1+2c+c^2+d^2}}\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-d + \sqrt{1+2c+c^2+d^2} + (1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} - \\
 & \frac{i(1+c) \operatorname{Log}\left[\frac{(1+c)(1+i \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right])}{1+c+i d-i \sqrt{1+2c+c^2+d^2}}\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-d + \sqrt{1+2c+c^2+d^2} + (1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} - \\
 & \frac{i(1+c) \operatorname{Log}\left[1 - \frac{-d + \sqrt{1+2c+c^2+d^2} + (1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{-i-i c-d+\sqrt{1+2c+c^2+d^2}}\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-d + \sqrt{1+2c+c^2+d^2} + (1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} + \\
 & \frac{i(1+c) \operatorname{Log}\left[1 - \frac{-d + \sqrt{1+2c+c^2+d^2} + (1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{i+i c-d+\sqrt{1+2c+c^2+d^2}}\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-d + \sqrt{1+2c+c^2+d^2} + (1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} + \\
 & \left. \begin{aligned}
 & \left(a \operatorname{Cos}\left[\frac{1}{2}(a+bx)\right]\right)^2 \left(-\operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]\right)^2 (d \operatorname{Cos}[a+bx] - (-1+c) \operatorname{Sin}[a+bx]) - \\
 & \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \left((-1+c) \operatorname{Cos}[a+bx] + d \operatorname{Sin}[a+bx]\right) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right] \Big) \Big) / \\
 & \left((-1+c) \operatorname{Cos}[a+bx] + d \operatorname{Sin}[a+bx]\right) + \\
 & \left(a \operatorname{Cos}\left[\frac{1}{2}(a+bx)\right]\right)^2 \left(\operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]\right)^2 (d \operatorname{Cos}[a+bx] - \operatorname{Sin}[a+bx] - c \operatorname{Sin}[a+bx]) + \\
 & \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 (\operatorname{Cos}[a+bx] + c \operatorname{Cos}[a+bx] + d \operatorname{Sin}[a+bx]) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right] \Big) \Big) / \\
 & \left. \left(\operatorname{Cos}[a+bx] + c \operatorname{Cos}[a+bx] + d \operatorname{Sin}[a+bx]\right) \right)
 \end{aligned}
 \right\}
 \end{aligned}$$

Problem 248: Result more than twice size of optimal antiderivative.

$$\int (e + f x)^3 \operatorname{ArcCoth}[\operatorname{Cot}[a + b x]] dx$$

Optimal (type 4, 302 leaves, 12 steps):

$$\begin{aligned} & \frac{(e + f x)^4 \operatorname{ArcCoth}[\operatorname{Cot}[a + b x]]}{4 f} + \frac{i (e + f x)^4 \operatorname{ArcTan}[e^{2 i (a + b x)}]}{4 f} - \\ & \frac{i (e + f x)^3 \operatorname{PolyLog}[2, -i e^{2 i (a + b x)}]}{4 b} + \frac{i (e + f x)^3 \operatorname{PolyLog}[2, i e^{2 i (a + b x)}]}{4 b} + \\ & \frac{3 f (e + f x)^2 \operatorname{PolyLog}[3, -i e^{2 i (a + b x)}]}{8 b^2} - \frac{3 f (e + f x)^2 \operatorname{PolyLog}[3, i e^{2 i (a + b x)}]}{8 b^2} + \\ & \frac{3 i f^2 (e + f x) \operatorname{PolyLog}[4, -i e^{2 i (a + b x)}]}{8 b^3} - \frac{3 i f^2 (e + f x) \operatorname{PolyLog}[4, i e^{2 i (a + b x)}]}{8 b^3} - \\ & \frac{3 f^3 \operatorname{PolyLog}[5, -i e^{2 i (a + b x)}]}{16 b^4} + \frac{3 f^3 \operatorname{PolyLog}[5, i e^{2 i (a + b x)}]}{16 b^4} \end{aligned}$$

Result (type 4, 654 leaves):

$$\begin{aligned} & \frac{1}{4} x (4 e^3 + 6 e^2 f x + 4 e f^2 x^2 + f^3 x^3) \operatorname{ArcCoth}[\operatorname{Cot}[a + b x]] + \\ & \frac{1}{16 b^4} \left( -8 b^4 e^3 x \operatorname{Log}[1 - i e^{2 i (a + b x)}] - 12 b^4 e^2 f x^2 \operatorname{Log}[1 - i e^{2 i (a + b x)}] - \right. \\ & \quad 8 b^4 e f^2 x^3 \operatorname{Log}[1 - i e^{2 i (a + b x)}] - 2 b^4 f^3 x^4 \operatorname{Log}[1 - i e^{2 i (a + b x)}] + 8 b^4 e^3 x \operatorname{Log}[1 + i e^{2 i (a + b x)}] + \\ & \quad 12 b^4 e^2 f x^2 \operatorname{Log}[1 + i e^{2 i (a + b x)}] + 8 b^4 e f^2 x^3 \operatorname{Log}[1 + i e^{2 i (a + b x)}] + \\ & \quad 2 b^4 f^3 x^4 \operatorname{Log}[1 + i e^{2 i (a + b x)}] - 4 i b^3 (e + f x)^3 \operatorname{PolyLog}[2, -i e^{2 i (a + b x)}] + \\ & \quad 4 i b^3 (e + f x)^3 \operatorname{PolyLog}[2, i e^{2 i (a + b x)}] + 6 b^2 e^2 f \operatorname{PolyLog}[3, -i e^{2 i (a + b x)}] + \\ & \quad 12 b^2 e f^2 x \operatorname{PolyLog}[3, -i e^{2 i (a + b x)}] + 6 b^2 f^3 x^2 \operatorname{PolyLog}[3, -i e^{2 i (a + b x)}] - \\ & \quad 6 b^2 e^2 f \operatorname{PolyLog}[3, i e^{2 i (a + b x)}] - 12 b^2 e f^2 x \operatorname{PolyLog}[3, i e^{2 i (a + b x)}] - \\ & \quad 6 b^2 f^3 x^2 \operatorname{PolyLog}[3, i e^{2 i (a + b x)}] + 6 i b e f^2 \operatorname{PolyLog}[4, -i e^{2 i (a + b x)}] + \\ & \quad 6 i b f^3 x \operatorname{PolyLog}[4, -i e^{2 i (a + b x)}] - 6 i b e f^2 \operatorname{PolyLog}[4, i e^{2 i (a + b x)}] - \\ & \quad \left. 6 i b f^3 x \operatorname{PolyLog}[4, i e^{2 i (a + b x)}] - 3 f^3 \operatorname{PolyLog}[5, -i e^{2 i (a + b x)}] + 3 f^3 \operatorname{PolyLog}[5, i e^{2 i (a + b x)}] \right) \end{aligned}$$

**Problem 255: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{ArcCoth}[c + d \operatorname{Cot}[a + b x]] dx$$

Optimal (type 4, 194 leaves, 7 steps):

$$\begin{aligned} & x \operatorname{ArcCoth}[c + d \operatorname{Cot}[a + b x]] + \\ & \frac{1}{2} x \operatorname{Log}\left[1 - \frac{(1 - c - i d) e^{2 i a + 2 i b x}}{1 - c + i d}\right] - \frac{1}{2} x \operatorname{Log}\left[1 - \frac{(1 + c + i d) e^{2 i a + 2 i b x}}{1 + c - i d}\right] - \\ & \frac{i \operatorname{PolyLog}\left[2, \frac{(1 - c - i d) e^{2 i a + 2 i b x}}{1 - c + i d}\right]}{4 b} + \frac{i \operatorname{PolyLog}\left[2, \frac{(1 + c + i d) e^{2 i a + 2 i b x}}{1 + c - i d}\right]}{4 b} \end{aligned}$$

Result (type 4, 4463 leaves):

$$\begin{aligned} & x \operatorname{ArcCoth}[c + d \operatorname{Cot}[a + b x]] - \\ & \left( d \left( a \operatorname{Log}\left[-\operatorname{Sec}\left[\frac{1}{2}(a + b x)\right]\right]^2 (d \operatorname{Cos}[a + b x] + (-1 + c) \operatorname{Sin}[a + b x]) \right) - \right. \end{aligned}$$

$$\begin{aligned}
& a \operatorname{Log}\left[-\operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2(d\cos[a+bx]+\sin[a+bx]+c\sin[a+bx])\right]- \\
& (a+bx)\operatorname{Log}\left[-\frac{-1+c+\sqrt{1-2c+c^2+d^2}}{d}+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right]- \\
& i\operatorname{Log}\left[\frac{d(-i+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right])}{-1+c-i d+\sqrt{1-2c+c^2+d^2}}\right]\operatorname{Log}\left[-\frac{-1+c+\sqrt{1-2c+c^2+d^2}}{d}+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right]+ \\
& i\operatorname{Log}\left[\frac{d(i+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right])}{-1+c+i d+\sqrt{1-2c+c^2+d^2}}\right]\operatorname{Log}\left[-\frac{-1+c+\sqrt{1-2c+c^2+d^2}}{d}+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right]+ \\
& (a+bx)\operatorname{Log}\left[-\frac{1+c+\sqrt{1+2c+c^2+d^2}}{d}+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right]+ \\
& i\operatorname{Log}\left[\frac{d(-i+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right])}{1+c-i d+\sqrt{1+2c+c^2+d^2}}\right]\operatorname{Log}\left[-\frac{1+c+\sqrt{1+2c+c^2+d^2}}{d}+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right]- \\
& i\operatorname{Log}\left[\frac{d(i+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right])}{1+c+i d+\sqrt{1+2c+c^2+d^2}}\right]\operatorname{Log}\left[-\frac{1+c+\sqrt{1+2c+c^2+d^2}}{d}+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right]- \\
& (a+bx)\operatorname{Log}\left[\frac{1-c+\sqrt{1-2c+c^2+d^2}+d\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{d}\right]- \\
& i\operatorname{Log}\left[-\frac{d(-i+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right])}{1-c+i d+\sqrt{1-2c+c^2+d^2}}\right]\operatorname{Log}\left[\frac{1-c+\sqrt{1-2c+c^2+d^2}+d\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{d}\right]+ \\
& i\operatorname{Log}\left[-\frac{d(i+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right])}{1-c-i d+\sqrt{1-2c+c^2+d^2}}\right]\operatorname{Log}\left[\frac{1-c+\sqrt{1-2c+c^2+d^2}+d\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{d}\right]+ \\
& (a+bx)\operatorname{Log}\left[\frac{-1-c+\sqrt{1+2c+c^2+d^2}+d\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{d}\right]+ \\
& i\operatorname{Log}\left[-\frac{d(-i+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right])}{-1-c+i d+\sqrt{1+2c+c^2+d^2}}\right]\operatorname{Log}\left[\frac{-1-c+\sqrt{1+2c+c^2+d^2}+d\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{d}\right]- \\
& i\operatorname{Log}\left[-\frac{d(i+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right])}{-1-c-i d+\sqrt{1+2c+c^2+d^2}}\right]\operatorname{Log}\left[\frac{-1-c+\sqrt{1+2c+c^2+d^2}+d\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{d}\right]- \\
& i\operatorname{PolyLog}\left[2,\frac{-1+c+\sqrt{1-2c+c^2+d^2}-d\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{-1+c-i d+\sqrt{1-2c+c^2+d^2}}\right]+ \\
& i\operatorname{PolyLog}\left[2,\frac{-1+c+\sqrt{1-2c+c^2+d^2}-d\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{-1+c+i d+\sqrt{1-2c+c^2+d^2}}\right]- \\
& i\operatorname{PolyLog}\left[2,\frac{1+c-\sqrt{1+2c+c^2+d^2}-d\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{1+c+i d-\sqrt{1+2c+c^2+d^2}}\right]+ \\
& i\operatorname{PolyLog}\left[2,\frac{1+c+\sqrt{1+2c+c^2+d^2}-d\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{1+c-i d+\sqrt{1+2c+c^2+d^2}}\right]-
\end{aligned}$$



$$\begin{aligned}
 & \left( \begin{aligned}
 & \text{i PolyLog}\left[2, \frac{1+c+\sqrt{1+2c+c^2+d^2}-d \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{1+c+i d+\sqrt{1+2c+c^2+d^2}}\right] + \\
 & \text{i PolyLog}\left[2, \frac{1-c+\sqrt{1-2c+c^2+d^2}+d \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{1-c-i d+\sqrt{1-2c+c^2+d^2}}\right] - \\
 & \text{i PolyLog}\left[2, \frac{1-c+\sqrt{1-2c+c^2+d^2}+d \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{1-c+i d+\sqrt{1-2c+c^2+d^2}}\right] + \\
 & \text{i PolyLog}\left[2, \frac{-1-c+\sqrt{1+2c+c^2+d^2}+d \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{-1-c+i d+\sqrt{1+2c+c^2+d^2}}\right] \right) \\
 & \left( (2a) / (b(1-c^2-d^2-\cos[2(a+bx)] + c^2 \cos[2(a+bx)] - d^2 \cos[2(a+bx)] - 2cd \sin[2(a+bx)])) - (2(a+bx)) / (b(1-c^2-d^2-\cos[2(a+bx)] + c^2 \cos[2(a+bx)] - d^2 \cos[2(a+bx)] - 2cd \sin[2(a+bx)])) \right) \Big/ \\
 & \left( \begin{aligned}
 & -\operatorname{Log}\left[-\frac{-1+c+\sqrt{1-2c+c^2+d^2}}{d} + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right] + \\
 & \operatorname{Log}\left[-\frac{1+c+\sqrt{1+2c+c^2+d^2}}{d} + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right] - \\
 & \operatorname{Log}\left[\frac{1-c+\sqrt{1-2c+c^2+d^2}+d \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{d}\right] + \\
 & \operatorname{Log}\left[\frac{-1-c+\sqrt{1+2c+c^2+d^2}+d \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{d}\right] - \\
 & \frac{\text{i Log}\left[-\frac{-1+c+\sqrt{1-2c+c^2+d^2}}{d} + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2(-i + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right])} + \\
 & \frac{\text{i Log}\left[-\frac{1+c+\sqrt{1+2c+c^2+d^2}}{d} + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2(-i + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right])} - \\
 & \frac{\text{i Log}\left[\frac{1-c+\sqrt{1-2c+c^2+d^2}+d \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{d}\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2(-i + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right])} + \\
 & \frac{\text{i Log}\left[\frac{-1-c+\sqrt{1+2c+c^2+d^2}+d \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{d}\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2(-i + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right])} +
 \end{aligned} \right)
 \end{aligned}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{i \operatorname{Log}\left[-\frac{-1+c+\sqrt{1-2c+c^2+d^2}}{d} + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(i + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} - \\
 & \frac{i \operatorname{Log}\left[-\frac{1+c+\sqrt{1+2c+c^2+d^2}}{d} + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(i + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} + \\
 & \frac{i \operatorname{Log}\left[\frac{1-c+\sqrt{1-2c+c^2+d^2} + d \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{d}\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(i + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} - \\
 & \frac{i \operatorname{Log}\left[\frac{-1-c+\sqrt{1+2c+c^2+d^2} + d \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{d}\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(i + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} - \\
 & \frac{(a+bx) \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-\frac{-1+c+\sqrt{1-2c+c^2+d^2}}{d} + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} - \\
 & \frac{i \operatorname{Log}\left[\frac{d\left(-i + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)}{-1+c-i d + \sqrt{1-2c+c^2+d^2}}\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-\frac{-1+c+\sqrt{1-2c+c^2+d^2}}{d} + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} + \\
 & \frac{i \operatorname{Log}\left[\frac{d\left(i + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)}{-1+c+i d + \sqrt{1-2c+c^2+d^2}}\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-\frac{-1+c+\sqrt{1-2c+c^2+d^2}}{d} + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} + \\
 & \frac{(a+bx) \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-\frac{1+c+\sqrt{1+2c+c^2+d^2}}{d} + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} + \frac{i \operatorname{Log}\left[\frac{d\left(-i + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)}{1+c-i d + \sqrt{1+2c+c^2+d^2}}\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-\frac{1+c+\sqrt{1+2c+c^2+d^2}}{d} + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} - \\
 & \frac{i \operatorname{Log}\left[\frac{d\left(i + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)}{1+c+i d + \sqrt{1+2c+c^2+d^2}}\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-\frac{1+c+\sqrt{1+2c+c^2+d^2}}{d} + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} - \\
 & \frac{i d \operatorname{Log}\left[1 - \frac{-1+c+\sqrt{1-2c+c^2+d^2} - d \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{-1+c-i d + \sqrt{1-2c+c^2+d^2}}\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-1+c+\sqrt{1-2c+c^2+d^2} - d \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} +
 \end{aligned}$$

$$\begin{aligned}
 & \frac{i \, d \, \text{Log} \left[ 1 - \frac{-1+c+\sqrt{1-2c+c^2+d^2} - d \, \text{Tan} \left[ \frac{1}{2} (a+bx) \right]}{-1+c+i \, d+\sqrt{1-2c+c^2+d^2}} \right] \, \text{Sec} \left[ \frac{1}{2} (a+bx) \right]^2}{2 \left( -1+c+\sqrt{1-2c+c^2+d^2} - d \, \text{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)} - \\
 & \frac{i \, d \, \text{Log} \left[ 1 - \frac{1+c-\sqrt{1+2c+c^2+d^2} - d \, \text{Tan} \left[ \frac{1}{2} (a+bx) \right]}{1+c+i \, d-\sqrt{1+2c+c^2+d^2}} \right] \, \text{Sec} \left[ \frac{1}{2} (a+bx) \right]^2}{2 \left( 1+c-\sqrt{1+2c+c^2+d^2} - d \, \text{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)} + \\
 & \frac{i \, d \, \text{Log} \left[ 1 - \frac{1+c+\sqrt{1+2c+c^2+d^2} - d \, \text{Tan} \left[ \frac{1}{2} (a+bx) \right]}{1+c-i \, d+\sqrt{1+2c+c^2+d^2}} \right] \, \text{Sec} \left[ \frac{1}{2} (a+bx) \right]^2}{2 \left( 1+c+\sqrt{1+2c+c^2+d^2} - d \, \text{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)} - \\
 & \frac{i \, d \, \text{Log} \left[ 1 - \frac{1+c+\sqrt{1+2c+c^2+d^2} - d \, \text{Tan} \left[ \frac{1}{2} (a+bx) \right]}{1+c+i \, d+\sqrt{1+2c+c^2+d^2}} \right] \, \text{Sec} \left[ \frac{1}{2} (a+bx) \right]^2}{2 \left( 1+c+\sqrt{1+2c+c^2+d^2} - d \, \text{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)} - \\
 & \frac{d (a+bx) \, \text{Sec} \left[ \frac{1}{2} (a+bx) \right]^2}{2 \left( 1-c+\sqrt{1-2c+c^2+d^2} + d \, \text{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)} - \\
 & \frac{i \, d \, \text{Log} \left[ -\frac{d \left( -i+\text{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)}{1-c+i \, d+\sqrt{1-2c+c^2+d^2}} \right] \, \text{Sec} \left[ \frac{1}{2} (a+bx) \right]^2}{2 \left( 1-c+\sqrt{1-2c+c^2+d^2} + d \, \text{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)} + \\
 & \frac{i \, d \, \text{Log} \left[ -\frac{d \left( i+\text{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)}{1-c-i \, d+\sqrt{1-2c+c^2+d^2}} \right] \, \text{Sec} \left[ \frac{1}{2} (a+bx) \right]^2}{2 \left( 1-c+\sqrt{1-2c+c^2+d^2} + d \, \text{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)} - \\
 & \frac{i \, d \, \text{Log} \left[ 1 - \frac{1-c+\sqrt{1-2c+c^2+d^2} + d \, \text{Tan} \left[ \frac{1}{2} (a+bx) \right]}{1-c-i \, d+\sqrt{1-2c+c^2+d^2}} \right] \, \text{Sec} \left[ \frac{1}{2} (a+bx) \right]^2}{2 \left( 1-c+\sqrt{1-2c+c^2+d^2} + d \, \text{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)} + \\
 & \frac{i \, d \, \text{Log} \left[ 1 - \frac{1-c+\sqrt{1-2c+c^2+d^2} + d \, \text{Tan} \left[ \frac{1}{2} (a+bx) \right]}{1-c+i \, d+\sqrt{1-2c+c^2+d^2}} \right] \, \text{Sec} \left[ \frac{1}{2} (a+bx) \right]^2}{2 \left( 1-c+\sqrt{1-2c+c^2+d^2} + d \, \text{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)} + \\
 & \frac{d (a+bx) \, \text{Sec} \left[ \frac{1}{2} (a+bx) \right]^2}{2 \left( -1-c+\sqrt{1+2c+c^2+d^2} + d \, \text{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)} + \\
 & \frac{i \, d \, \text{Log} \left[ -\frac{d \left( -i+\text{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)}{-1-c+i \, d+\sqrt{1+2c+c^2+d^2}} \right] \, \text{Sec} \left[ \frac{1}{2} (a+bx) \right]^2}{2 \left( -1-c+\sqrt{1+2c+c^2+d^2} + d \, \text{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)} -
 \end{aligned}$$

$$\begin{aligned}
 & \frac{i d \operatorname{Log}\left[-\frac{d\left(i+\operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}{-1-c-i d+\sqrt{1+2 c+c^2+d^2}}\right] \operatorname{Sec}\left[\frac{1}{2}(a+b x)\right]^2}{2\left(-1-c+\sqrt{1+2 c+c^2+d^2}+d \operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]\right)} - \\
 & \frac{i d \operatorname{Log}\left[1-\frac{-1-c+\sqrt{1+2 c+c^2+d^2}+d \operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]}{-1-c+i d+\sqrt{1+2 c+c^2+d^2}}\right] \operatorname{Sec}\left[\frac{1}{2}(a+b x)\right]^2}{2\left(-1-c+\sqrt{1+2 c+c^2+d^2}+d \operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]\right)} - \\
 & \left(a \operatorname{Cos}\left[\frac{1}{2}(a+b x)\right]\right)^2\left(-\operatorname{Sec}\left[\frac{1}{2}(a+b x)\right]\right)^2\left((-1+c) \operatorname{Cos}[a+b x]-d \operatorname{Sin}[a+b x]\right)- \\
 & \quad \operatorname{Sec}\left[\frac{1}{2}(a+b x)\right]^2\left(d \operatorname{Cos}[a+b x]+(-1+c) \operatorname{Sin}[a+b x]\right) \operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]\right) / / \\
 & \quad \left(d \operatorname{Cos}[a+b x]+(-1+c) \operatorname{Sin}[a+b x]\right)+ \\
 & \left(a \operatorname{Cos}\left[\frac{1}{2}(a+b x)\right]\right)^2\left(-\operatorname{Sec}\left[\frac{1}{2}(a+b x)\right]\right)^2\left(\operatorname{Cos}[a+b x]+c \operatorname{Cos}[a+b x]-d \operatorname{Sin}[a+b x]\right)- \\
 & \quad \operatorname{Sec}\left[\frac{1}{2}(a+b x)\right]^2\left(d \operatorname{Cos}[a+b x]+\operatorname{Sin}[a+b x]+c \operatorname{Sin}[a+b x]\right) \operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]\right) / / \\
 & \quad \left(d \operatorname{Cos}[a+b x]+\operatorname{Sin}[a+b x]+c \operatorname{Sin}[a+b x]\right)
 \end{aligned}$$

**Problem 265: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+b \operatorname{ArcCoth}[c x^n]) (d+e \operatorname{Log}[f x^m])}{x} dx$$

Optimal (type 4, 160 leaves, 11 steps):

$$\begin{aligned}
 & a d \operatorname{Log}[x] + \frac{a e \operatorname{Log}[f x^m]^2}{2 m} + \frac{b d \operatorname{PolyLog}\left[2,-\frac{x^n}{c}\right]}{2 n} + \\
 & \frac{b e \operatorname{Log}[f x^m] \operatorname{PolyLog}\left[2,-\frac{x^n}{c}\right]}{2 n} - \frac{b d \operatorname{PolyLog}\left[2,\frac{x^n}{c}\right]}{2 n} - \\
 & \frac{b e \operatorname{Log}[f x^m] \operatorname{PolyLog}\left[2,\frac{x^n}{c}\right]}{2 n} + \frac{b e m \operatorname{PolyLog}\left[3,-\frac{x^n}{c}\right]}{2 n^2} - \frac{b e m \operatorname{PolyLog}\left[3,\frac{x^n}{c}\right]}{2 n^2}
 \end{aligned}$$

Result (type 5, 131 leaves):

$$\begin{aligned}
 & -\frac{b c e m x^n \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 1\right\},\left\{\frac{3}{2}, \frac{3}{2}, \frac{3}{2}\right\}, c^2 x^{2 n}\right]}{n^2} + \frac{1}{n} \\
 & b c x^n \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{2}, \frac{1}{2}, 1\right\},\left\{\frac{3}{2}, \frac{3}{2}\right\}, c^2 x^{2 n}\right] (d+e \operatorname{Log}[f x^m]) - \\
 & \frac{1}{2}(a+b \operatorname{ArcCoth}[c x^n]-b \operatorname{ArcTanh}[c x^n]) \operatorname{Log}[x] (e m \operatorname{Log}[x]-2(d+e \operatorname{Log}[f x^m]))
 \end{aligned}$$

### Problem 269: Unable to integrate problem.

$$\int \frac{(a + b \operatorname{ArcCoth}[c x]) (d + e \operatorname{Log}[1 - c^2 x^2])}{x} dx$$

Optimal (type 4, 381 leaves, 21 steps):

$$\begin{aligned} & -\frac{1}{2} b e \operatorname{Log}\left[1 + \frac{1}{c x}\right]^2 \operatorname{Log}\left[-\frac{1}{c x}\right] + \frac{1}{2} b e \operatorname{Log}\left[1 - \frac{1}{c x}\right]^2 \operatorname{Log}\left[\frac{1}{c x}\right] + a d \operatorname{Log}[x] - \\ & b e \operatorname{Log}\left[\frac{c + \frac{1}{x}}{c}\right] \operatorname{PolyLog}\left[2, \frac{c + \frac{1}{x}}{c}\right] + b e \operatorname{Log}\left[1 - \frac{1}{c x}\right] \operatorname{PolyLog}\left[2, 1 - \frac{1}{c x}\right] + \\ & \frac{1}{2} b d \operatorname{PolyLog}\left[2, -\frac{1}{c x}\right] + \frac{1}{2} b e \operatorname{Log}\left[-c^2 x^2\right] \operatorname{PolyLog}\left[2, -\frac{1}{c x}\right] - \\ & \frac{1}{2} b e \left(\operatorname{Log}\left[1 - \frac{1}{c x}\right] + \operatorname{Log}\left[1 + \frac{1}{c x}\right] + \operatorname{Log}\left[-c^2 x^2\right] - \operatorname{Log}\left[1 - c^2 x^2\right]\right) \operatorname{PolyLog}\left[2, -\frac{1}{c x}\right] - \\ & \frac{1}{2} b d \operatorname{PolyLog}\left[2, \frac{1}{c x}\right] - \frac{1}{2} b e \operatorname{Log}\left[-c^2 x^2\right] \operatorname{PolyLog}\left[2, \frac{1}{c x}\right] + \\ & \frac{1}{2} b e \left(\operatorname{Log}\left[1 - \frac{1}{c x}\right] + \operatorname{Log}\left[1 + \frac{1}{c x}\right] + \operatorname{Log}\left[-c^2 x^2\right] - \operatorname{Log}\left[1 - c^2 x^2\right]\right) \operatorname{PolyLog}\left[2, \frac{1}{c x}\right] - \\ & \frac{1}{2} a e \operatorname{PolyLog}\left[2, c^2 x^2\right] + b e \operatorname{PolyLog}\left[3, \frac{c + \frac{1}{x}}{c}\right] - \\ & b e \operatorname{PolyLog}\left[3, 1 - \frac{1}{c x}\right] + b e \operatorname{PolyLog}\left[3, -\frac{1}{c x}\right] - b e \operatorname{PolyLog}\left[3, \frac{1}{c x}\right] \end{aligned}$$

Result (type 8, 29 leaves):

$$\int \frac{(a + b \operatorname{ArcCoth}[c x]) (d + e \operatorname{Log}[1 - c^2 x^2])}{x} dx$$

### Problem 275: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b \operatorname{ArcCoth}[c x]) (d + e \operatorname{Log}[1 - c^2 x^2])}{x^2} dx$$

Optimal (type 4, 105 leaves, 6 steps):

$$\begin{aligned} & -\frac{c e (a + b \operatorname{ArcCoth}[c x])^2}{b} - \frac{(a + b \operatorname{ArcCoth}[c x]) (d + e \operatorname{Log}[1 - c^2 x^2])}{x} + \\ & \frac{1}{2} b c (d + e \operatorname{Log}[1 - c^2 x^2]) \operatorname{Log}\left[1 - \frac{1}{1 - c^2 x^2}\right] - \frac{1}{2} b c e \operatorname{PolyLog}\left[2, \frac{1}{1 - c^2 x^2}\right] \end{aligned}$$

Result (type 4, 332 leaves):

$$\begin{aligned}
& -\frac{1}{4x} \left( 4ad + 4bd \operatorname{ArcCoth}[cx] + 4bcex \operatorname{ArcCoth}[cx]^2 + \right. \\
& \quad 8acex \operatorname{ArcTanh}[cx] - 4bcdx \operatorname{Log}[x] - bcex \operatorname{Log}\left[-\frac{1}{c} + x\right]^2 - bcex \operatorname{Log}\left[\frac{1}{c} + x\right]^2 - \\
& \quad 2bcex \operatorname{Log}\left[\frac{1}{c} + x\right] \operatorname{Log}\left[\frac{1}{2}(1 - cx)\right] + 4bcex \operatorname{Log}[x] \operatorname{Log}[1 - cx] - \\
& \quad 2bcex \operatorname{Log}\left[-\frac{1}{c} + x\right] \operatorname{Log}\left[\frac{1}{2}(1 + cx)\right] + 4bcex \operatorname{Log}[x] \operatorname{Log}[1 + cx] + \\
& \quad 4ae \operatorname{Log}[1 - c^2x^2] + 2bcdx \operatorname{Log}[1 - c^2x^2] + 4be \operatorname{ArcCoth}[cx] \operatorname{Log}[1 - c^2x^2] - \\
& \quad 4bcex \operatorname{Log}[x] \operatorname{Log}[1 - c^2x^2] + 2bcex \operatorname{Log}\left[-\frac{1}{c} + x\right] \operatorname{Log}[1 - c^2x^2] + \\
& \quad 2bcex \operatorname{Log}\left[\frac{1}{c} + x\right] \operatorname{Log}[1 - c^2x^2] + 4bcex \operatorname{PolyLog}[2, -cx] + 4bcex \operatorname{PolyLog}[2, cx] - \\
& \quad \left. 2bcex \operatorname{PolyLog}\left[2, \frac{1}{2} - \frac{cx}{2}\right] - 2bcex \operatorname{PolyLog}\left[2, \frac{1}{2}(1 + cx)\right] \right)
\end{aligned}$$

**Problem 276: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \operatorname{ArcCoth}[cx]) (d + e \operatorname{Log}[1 - c^2x^2])}{x^4} dx$$

Optimal (type 4, 197 leaves, 15 steps):

$$\begin{aligned}
& \frac{2c^2e(a + b \operatorname{ArcCoth}[cx])}{3x} - \frac{c^3e(a + b \operatorname{ArcCoth}[cx])^2}{3b} - bc^3e \operatorname{Log}[x] + \frac{1}{3}bc^3e \operatorname{Log}[1 - c^2x^2] - \\
& \frac{bc(1 - c^2x^2)(d + e \operatorname{Log}[1 - c^2x^2])}{6x^2} - \frac{(a + b \operatorname{ArcCoth}[cx])(d + e \operatorname{Log}[1 - c^2x^2])}{3x^3} + \\
& \frac{1}{6}bc^3(d + e \operatorname{Log}[1 - c^2x^2]) \operatorname{Log}\left[1 - \frac{1}{1 - c^2x^2}\right] - \frac{1}{6}bc^3e \operatorname{PolyLog}\left[2, \frac{1}{1 - c^2x^2}\right]
\end{aligned}$$

Result (type 4, 457 leaves):

$$\frac{1}{6} \left( -\frac{2 a d}{x^3} - \frac{b c d}{x^2} + \frac{4 a c^2 e}{x} - \frac{2 b d \operatorname{ArcCoth}[c x]}{x^3} + \frac{4 b c^2 e \operatorname{ArcCoth}[c x]}{x} - 2 b c^3 e \operatorname{ArcCoth}[c x]^2 - \right.$$

$$4 a c^3 e \operatorname{ArcTanh}[c x] - 4 b c^3 e \operatorname{Log}\left[\frac{1}{\sqrt{1 - \frac{1}{c^2 x^2}}}\right] + 2 b c^3 d \operatorname{Log}[x] - 2 b c^3 e \operatorname{Log}[x] +$$

$$\frac{1}{2} b c^3 e \operatorname{Log}\left[-\frac{1}{c} + x\right]^2 + \frac{1}{2} b c^3 e \operatorname{Log}\left[\frac{1}{c} + x\right]^2 + b c^3 e \operatorname{Log}\left[\frac{1}{c} + x\right] \operatorname{Log}\left[\frac{1}{2}(1 - c x)\right] -$$

$$2 b c^3 e \operatorname{Log}[x] \operatorname{Log}[1 - c x] + b c^3 e \operatorname{Log}\left[-\frac{1}{c} + x\right] \operatorname{Log}\left[\frac{1}{2}(1 + c x)\right] -$$

$$2 b c^3 e \operatorname{Log}[x] \operatorname{Log}[1 + c x] - b c^3 d \operatorname{Log}[1 - c^2 x^2] + b c^3 e \operatorname{Log}[1 - c^2 x^2] - \frac{2 a e \operatorname{Log}[1 - c^2 x^2]}{x^3} -$$

$$\frac{b c e \operatorname{Log}[1 - c^2 x^2]}{x^2} - \frac{2 b e \operatorname{ArcCoth}[c x] \operatorname{Log}[1 - c^2 x^2]}{x^3} + 2 b c^3 e \operatorname{Log}[x] \operatorname{Log}[1 - c^2 x^2] -$$

$$b c^3 e \operatorname{Log}\left[-\frac{1}{c} + x\right] \operatorname{Log}[1 - c^2 x^2] - b c^3 e \operatorname{Log}\left[\frac{1}{c} + x\right] \operatorname{Log}[1 - c^2 x^2] - 2 b c^3 e \operatorname{PolyLog}[2, -c x] -$$

$$\left. 2 b c^3 e \operatorname{PolyLog}[2, c x] + b c^3 e \operatorname{PolyLog}\left[2, \frac{1}{2} - \frac{c x}{2}\right] + b c^3 e \operatorname{PolyLog}\left[2, \frac{1}{2}(1 + c x)\right] \right)$$

### Problem 277: Unable to integrate problem.

$$\int \frac{(a + b \operatorname{ArcCoth}[c x]) (d + e \operatorname{Log}[1 - c^2 x^2])}{x^6} dx$$

Optimal (type 4, 256 leaves, 24 steps):

$$\frac{7 b c^3 e}{60 x^2} + \frac{2 c^2 e (a + b \operatorname{ArcCoth}[c x])}{15 x^3} + \frac{2 c^4 e (a + b \operatorname{ArcCoth}[c x])}{5 x} - \frac{c^5 e (a + b \operatorname{ArcCoth}[c x])^2}{5 b}$$

$$- \frac{5}{6} b c^5 e \operatorname{Log}[x] + \frac{19}{60} b c^5 e \operatorname{Log}[1 - c^2 x^2] - \frac{b c (d + e \operatorname{Log}[1 - c^2 x^2])}{20 x^4} -$$

$$+ \frac{b c^3 (1 - c^2 x^2) (d + e \operatorname{Log}[1 - c^2 x^2])}{10 x^2} - \frac{(a + b \operatorname{ArcCoth}[c x]) (d + e \operatorname{Log}[1 - c^2 x^2])}{5 x^5} +$$

$$- \frac{1}{10} b c^5 (d + e \operatorname{Log}[1 - c^2 x^2]) \operatorname{Log}\left[1 - \frac{1}{1 - c^2 x^2}\right] - \frac{1}{10} b c^5 e \operatorname{PolyLog}\left[2, \frac{1}{1 - c^2 x^2}\right]$$

Result (type 8, 29 leaves):

$$\int \frac{(a + b \operatorname{ArcCoth}[c x]) (d + e \operatorname{Log}[1 - c^2 x^2])}{x^6} dx$$

### Problem 278: Result unnecessarily involves complex numbers and more than

twice size of optimal antiderivative.

$$\int x (a + b \operatorname{ArcCoth}[c x]) (d + e \operatorname{Log}[f + g x^2]) dx$$

Optimal (type 4, 512 leaves, 22 steps):

$$\begin{aligned} & \frac{b(d-e)x}{2c} - \frac{bex}{c} + \frac{1}{2} dx^2 (a + b \operatorname{ArcCoth}[c x]) - \\ & \frac{1}{2} ex^2 (a + b \operatorname{ArcCoth}[c x]) + \frac{be\sqrt{f} \operatorname{ArcTan}\left[\frac{\sqrt{g}x}{\sqrt{f}}\right]}{c\sqrt{g}} - \frac{b(d-e) \operatorname{ArcTanh}[c x]}{2c^2} - \\ & \frac{be(c^2f+g) \operatorname{ArcTanh}[c x] \operatorname{Log}\left[\frac{2}{1+cx}\right]}{c^2g} + \frac{be(c^2f+g) \operatorname{ArcTanh}[c x] \operatorname{Log}\left[\frac{2c(\sqrt{-f}-\sqrt{g}x)}{(c\sqrt{-f}-\sqrt{g})(1+cx)}\right]}{2c^2g} + \\ & \frac{be(c^2f+g) \operatorname{ArcTanh}[c x] \operatorname{Log}\left[\frac{2c(\sqrt{-f}+\sqrt{g}x)}{(c\sqrt{-f}+\sqrt{g})(1+cx)}\right]}{2c^2g} + \\ & \frac{bex \operatorname{Log}[f+gx^2]}{2c} + \frac{e(f+gx^2)(a+b \operatorname{ArcCoth}[c x]) \operatorname{Log}[f+gx^2]}{2g} - \\ & \frac{be(c^2f+g) \operatorname{ArcTanh}[c x] \operatorname{Log}[f+gx^2]}{2c^2g} + \frac{be(c^2f+g) \operatorname{PolyLog}\left[2, 1-\frac{2}{1+cx}\right]}{2c^2g} - \\ & \frac{be(c^2f+g) \operatorname{PolyLog}\left[2, 1-\frac{2c(\sqrt{-f}-\sqrt{g}x)}{(c\sqrt{-f}-\sqrt{g})(1+cx)}\right]}{4c^2g} - \frac{be(c^2f+g) \operatorname{PolyLog}\left[2, 1-\frac{2c(\sqrt{-f}+\sqrt{g}x)}{(c\sqrt{-f}+\sqrt{g})(1+cx)}\right]}{4c^2g} \end{aligned}$$

Result (type 4, 1128 leaves):

$$\begin{aligned} & \frac{1}{4c^2g} \left( 2bcdgx - 6bcegx + 2ac^2dgx^2 - 2ac^2egx^2 - 2bdg \operatorname{ArcCoth}[c x] + \right. \\ & 2be g \operatorname{ArcCoth}[c x] + 2bc^2d g x^2 \operatorname{ArcCoth}[c x] - 2bc^2e g x^2 \operatorname{ArcCoth}[c x] + \\ & 4bce\sqrt{f}\sqrt{g} \operatorname{ArcTan}\left[\frac{\sqrt{g}x}{\sqrt{f}}\right] - 4ibc^2ef \operatorname{ArcSin}\left[\sqrt{\frac{g}{c^2f+g}}\right] \operatorname{ArcTanh}\left[\frac{cf}{\sqrt{-c^2fg}x}\right] - \\ & 4ibeg \operatorname{ArcSin}\left[\sqrt{\frac{g}{c^2f+g}}\right] \operatorname{ArcTanh}\left[\frac{cf}{\sqrt{-c^2fg}x}\right] - \\ & 4bc^2ef \operatorname{ArcCoth}[c x] \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcCoth}[c x]}\right] - \\ & 4be g \operatorname{ArcCoth}[c x] \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcCoth}[c x]}\right] + 2bc^2ef \operatorname{ArcCoth}[c x] \operatorname{Log}\left[\frac{1}{c^2f+g}\right. \\ & \left. e^{-2 \operatorname{ArcCoth}[c x]} \left( c^2(-1 + e^{2 \operatorname{ArcCoth}[c x]})f + g + e^{2 \operatorname{ArcCoth}[c x]}g - 2\sqrt{-c^2fg} \right) \right] + 2be g \operatorname{ArcCoth}[c x] \\ & \left. \operatorname{Log}\left[\frac{1}{c^2f+g} e^{-2 \operatorname{ArcCoth}[c x]} \left( c^2(-1 + e^{2 \operatorname{ArcCoth}[c x]})f + g + e^{2 \operatorname{ArcCoth}[c x]}g - 2\sqrt{-c^2fg} \right) \right] \right) - \end{aligned}$$



$$\begin{aligned}
 & 2 \operatorname{Im} b c^2 e f \operatorname{ArcSin} \left[ \sqrt{\frac{g}{c^2 f + g}} \right] \operatorname{Log} \left[ \frac{1}{c^2 f + g} e^{-2 \operatorname{ArcCoth}[c x]} \right. \\
 & \quad \left. \left( c^2 (-1 + e^{2 \operatorname{ArcCoth}[c x]}) f + g + e^{2 \operatorname{ArcCoth}[c x]} g - 2 \sqrt{-c^2 f g} \right) \right] - 2 \operatorname{Im} b e g \operatorname{ArcSin} \left[ \sqrt{\frac{g}{c^2 f + g}} \right] \\
 & \operatorname{Log} \left[ \frac{1}{c^2 f + g} e^{-2 \operatorname{ArcCoth}[c x]} \left( c^2 (-1 + e^{2 \operatorname{ArcCoth}[c x]}) f + g + e^{2 \operatorname{ArcCoth}[c x]} g - 2 \sqrt{-c^2 f g} \right) \right] + \\
 & 2 b c^2 e f \operatorname{ArcCoth}[c x] \operatorname{Log} \left[ \frac{1}{c^2 f + g} \right. \\
 & \quad \left. e^{-2 \operatorname{ArcCoth}[c x]} \left( c^2 (-1 + e^{2 \operatorname{ArcCoth}[c x]}) f + g + e^{2 \operatorname{ArcCoth}[c x]} g + 2 \sqrt{-c^2 f g} \right) \right] + 2 b e g \operatorname{ArcCoth}[c x] \\
 & \operatorname{Log} \left[ \frac{1}{c^2 f + g} e^{-2 \operatorname{ArcCoth}[c x]} \left( c^2 (-1 + e^{2 \operatorname{ArcCoth}[c x]}) f + g + e^{2 \operatorname{ArcCoth}[c x]} g + 2 \sqrt{-c^2 f g} \right) \right] + \\
 & 2 \operatorname{Im} b c^2 e f \operatorname{ArcSin} \left[ \sqrt{\frac{g}{c^2 f + g}} \right] \operatorname{Log} \left[ \frac{1}{c^2 f + g} e^{-2 \operatorname{ArcCoth}[c x]} \right. \\
 & \quad \left. \left( c^2 (-1 + e^{2 \operatorname{ArcCoth}[c x]}) f + g + e^{2 \operatorname{ArcCoth}[c x]} g + 2 \sqrt{-c^2 f g} \right) \right] + 2 \operatorname{Im} b e g \operatorname{ArcSin} \left[ \sqrt{\frac{g}{c^2 f + g}} \right] \\
 & \operatorname{Log} \left[ \frac{1}{c^2 f + g} e^{-2 \operatorname{ArcCoth}[c x]} \left( c^2 (-1 + e^{2 \operatorname{ArcCoth}[c x]}) f + g + e^{2 \operatorname{ArcCoth}[c x]} g + 2 \sqrt{-c^2 f g} \right) \right] + \\
 & 2 a c^2 e f \operatorname{Log}[f + g x^2] + 2 b c e g x \operatorname{Log}[f + g x^2] + 2 a c^2 e g x^2 \operatorname{Log}[f + g x^2] - \\
 & 2 b e g \operatorname{ArcCoth}[c x] \operatorname{Log}[f + g x^2] + 2 b c^2 e g x^2 \operatorname{ArcCoth}[c x] \operatorname{Log}[f + g x^2] + \\
 & 2 b e (c^2 f + g) \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcCoth}[c x]} \right] - \\
 & b e (c^2 f + g) \operatorname{PolyLog}\left[2, \frac{e^{-2 \operatorname{ArcCoth}[c x]} \left( c^2 f - g + 2 \sqrt{-c^2 f g} \right)}{c^2 f + g} \right] - \\
 & b c^2 e f \operatorname{PolyLog}\left[2, -\frac{e^{-2 \operatorname{ArcCoth}[c x]} \left( -c^2 f + g + 2 \sqrt{-c^2 f g} \right)}{c^2 f + g} \right] - \\
 & b e g \operatorname{PolyLog}\left[2, -\frac{e^{-2 \operatorname{ArcCoth}[c x]} \left( -c^2 f + g + 2 \sqrt{-c^2 f g} \right)}{c^2 f + g} \right] \Bigg)
 \end{aligned}$$

**Problem 279: Result more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{ArcCoth}[c x]) (d + e \operatorname{Log}[f + g x^2]) dx$$

Optimal (type 4, 546 leaves, 38 steps):

$$\begin{aligned}
 & -2 a e x - 2 b e x \operatorname{ArcCoth}[c x] + \frac{2 a e \sqrt{f} \operatorname{ArcTan}\left[\frac{\sqrt{g} x}{\sqrt{f}}\right]}{\sqrt{g}} - \frac{b e \sqrt{f} \operatorname{ArcTan}\left[\frac{\sqrt{g} x}{\sqrt{f}}\right] \operatorname{Log}\left[1 - \frac{1}{c x}\right]}{\sqrt{g}} + \\
 & \frac{b e \sqrt{f} \operatorname{ArcTan}\left[\frac{\sqrt{g} x}{\sqrt{f}}\right] \operatorname{Log}\left[1 + \frac{1}{c x}\right]}{\sqrt{g}} + \frac{b e \sqrt{f} \operatorname{ArcTan}\left[\frac{\sqrt{g} x}{\sqrt{f}}\right] \operatorname{Log}\left[-\frac{2 \sqrt{f} \sqrt{g} (1-c x)}{(i c \sqrt{f}-\sqrt{g})(\sqrt{f}-i \sqrt{g} x)}\right]}{\sqrt{g}} - \\
 & \frac{b e \sqrt{f} \operatorname{ArcTan}\left[\frac{\sqrt{g} x}{\sqrt{f}}\right] \operatorname{Log}\left[\frac{2 \sqrt{f} \sqrt{g} (1+c x)}{(i c \sqrt{f}+\sqrt{g})(\sqrt{f}-i \sqrt{g} x)}\right]}{\sqrt{g}} - \frac{b e \operatorname{Log}\left[1-c^2 x^2\right]}{c} + \\
 & x\left(a+b \operatorname{ArcCoth}[c x]\right)\left(d+e \operatorname{Log}\left[f+g x^2\right]\right) + \frac{b \operatorname{Log}\left[\frac{g\left(1-c^2 x^2\right)}{c^2 f+g}\right]\left(d+e \operatorname{Log}\left[f+g x^2\right]\right)}{2 c} + \\
 & \frac{b e \operatorname{PolyLog}\left[2, \frac{c^2\left(f+g x^2\right)}{c^2 f+g}\right]}{2 c} - \frac{i b e \sqrt{f} \operatorname{PolyLog}\left[2, 1+\frac{2 \sqrt{f} \sqrt{g} (1-c x)}{(i c \sqrt{f}-\sqrt{g})(\sqrt{f}-i \sqrt{g} x)}\right]}{2 \sqrt{g}} + \\
 & \frac{i b e \sqrt{f} \operatorname{PolyLog}\left[2, 1-\frac{2 \sqrt{f} \sqrt{g} (1+c x)}{(i c \sqrt{f}+\sqrt{g})(\sqrt{f}-i \sqrt{g} x)}\right]}{2 \sqrt{g}}
 \end{aligned}$$

Result (type 4, 1287 leaves):

$$\begin{aligned}
 & a d x - 2 a e x + b d x \operatorname{ArcCoth}[c x] + \frac{2 a e \sqrt{f} \operatorname{ArcTan}\left[\frac{\sqrt{g} x}{\sqrt{f}}\right]}{\sqrt{g}} + \frac{b d \operatorname{Log}\left[1-c^2 x^2\right]}{2 c} + \\
 & a e x \operatorname{Log}\left[f+g x^2\right] + b e\left(x \operatorname{ArcCoth}[c x] + \frac{\operatorname{Log}\left[1-c^2 x^2\right]}{2 c}\right) \operatorname{Log}\left[f+g x^2\right] + \\
 & \frac{1}{2 c} b e\left(-4 c x \operatorname{ArcCoth}[c x] + 4 \operatorname{Log}\left[\frac{1}{c \sqrt{1-\frac{1}{c^2 x^2}} x}\right]\right) + \\
 & \frac{1}{g} \sqrt{c^2 f g}\left(-2 i \operatorname{ArcCos}\left[\frac{c^2 f-g}{c^2 f+g}\right] \operatorname{ArcTan}\left[\frac{\sqrt{c^2 f g}}{c g x}\right] + 4 \operatorname{ArcCoth}[c x] \operatorname{ArcTan}\left[\frac{c g x}{\sqrt{c^2 f g}}\right] - \right. \\
 & \left. \left(\operatorname{ArcCos}\left[\frac{c^2 f-g}{c^2 f+g}\right] + 2 \operatorname{ArcTan}\left[\frac{\sqrt{c^2 f g}}{c g x}\right]\right) \operatorname{Log}\left[\frac{2 i g\left(i c^2 f+\sqrt{c^2 f g}\right)\left(-1+\frac{1}{c x}\right)}{\left(c^2 f+g\right)\left(g+\frac{i \sqrt{c^2 f g}}{c x}\right)}\right] - \right. \\
 & \left. \left(\operatorname{ArcCos}\left[\frac{c^2 f-g}{c^2 f+g}\right] - 2 \operatorname{ArcTan}\left[\frac{\sqrt{c^2 f g}}{c g x}\right]\right) \operatorname{Log}\left[\frac{2 g\left(c^2 f+i \sqrt{c^2 f g}\right)\left(1+\frac{1}{c x}\right)}{\left(c^2 f+g\right)\left(g+\frac{i \sqrt{c^2 f g}}{c x}\right)}\right] + \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left( \operatorname{ArcCos}\left[\frac{c^2 f - g}{c^2 f + g}\right] + 2 \left( \operatorname{ArcTan}\left[\frac{\sqrt{c^2 f g}}{c g x}\right] + \operatorname{ArcTan}\left[\frac{c g x}{\sqrt{c^2 f g}}\right] \right) \right) \\
 & \operatorname{Log}\left[\frac{\sqrt{2} e^{-\operatorname{ArcCoth}[c x]} \sqrt{c^2 f g}}{\sqrt{c^2 f + g} \sqrt{-c^2 f + g + (c^2 f + g) \operatorname{Cosh}[2 \operatorname{ArcCoth}[c x]]}}\right] + \\
 & \left( \operatorname{ArcCos}\left[\frac{c^2 f - g}{c^2 f + g}\right] - 2 \left( \operatorname{ArcTan}\left[\frac{\sqrt{c^2 f g}}{c g x}\right] + \operatorname{ArcTan}\left[\frac{c g x}{\sqrt{c^2 f g}}\right] \right) \right) \\
 & \operatorname{Log}\left[\frac{\sqrt{2} e^{\operatorname{ArcCoth}[c x]} \sqrt{c^2 f g}}{\sqrt{c^2 f + g} \sqrt{-c^2 f + g + (c^2 f + g) \operatorname{Cosh}[2 \operatorname{ArcCoth}[c x]]}}\right] + \\
 & i \left( -\operatorname{PolyLog}\left[2, \frac{(-c^2 f + g + 2 i \sqrt{c^2 f g}) \left(g - \frac{i \sqrt{c^2 f g}}{c x}\right)}{(c^2 f + g) \left(g + \frac{i \sqrt{c^2 f g}}{c x}\right)}\right] + \right. \\
 & \left. \operatorname{PolyLog}\left[2, \frac{(c^2 f - g + 2 i \sqrt{c^2 f g}) \left(i g + \frac{\sqrt{c^2 f g}}{c x}\right)}{(c^2 f + g) \left(-i g + \frac{\sqrt{c^2 f g}}{c x}\right)}\right] \right) \Bigg) - \\
 & \frac{1}{c} \operatorname{be} g \left( \frac{\left(-\operatorname{Log}\left[-\frac{1}{c} + x\right] - \operatorname{Log}\left[\frac{1}{c} + x\right] + \operatorname{Log}\left[1 - c^2 x^2\right]\right) \operatorname{Log}\left[f + g x^2\right]}{2 g} + \right. \\
 & \frac{\operatorname{Log}\left[-\frac{1}{c} + x\right] \operatorname{Log}\left[1 - \frac{\sqrt{g} \left(-\frac{1}{c} + x\right)}{-i \sqrt{f} - \frac{\sqrt{g}}{c}}\right] + \operatorname{PolyLog}\left[2, \frac{\sqrt{g} \left(-\frac{1}{c} + x\right)}{-i \sqrt{f} - \frac{\sqrt{g}}{c}}\right]}{2 g} + \\
 & \frac{\operatorname{Log}\left[-\frac{1}{c} + x\right] \operatorname{Log}\left[1 - \frac{\sqrt{g} \left(-\frac{1}{c} + x\right)}{i \sqrt{f} - \frac{\sqrt{g}}{c}}\right] + \operatorname{PolyLog}\left[2, \frac{\sqrt{g} \left(-\frac{1}{c} + x\right)}{i \sqrt{f} - \frac{\sqrt{g}}{c}}\right]}{2 g} + \\
 & \left. \frac{\operatorname{Log}\left[\frac{1}{c} + x\right] \operatorname{Log}\left[1 - \frac{\sqrt{g} \left(\frac{1}{c} + x\right)}{-i \sqrt{f} + \frac{\sqrt{g}}{c}}\right] + \operatorname{PolyLog}\left[2, \frac{\sqrt{g} \left(\frac{1}{c} + x\right)}{-i \sqrt{f} + \frac{\sqrt{g}}{c}}\right]}{2 g} + \right.
 \end{aligned}$$

$$\left. \frac{\text{Log}\left[\frac{1}{c} + x\right] \text{Log}\left[1 - \frac{\sqrt{g}\left(\frac{1}{c} + x\right)}{i\sqrt{f} + \frac{\sqrt{g}}{c}}\right] + \text{PolyLog}\left[2, \frac{\sqrt{g}\left(\frac{1}{c} + x\right)}{i\sqrt{f} + \frac{\sqrt{g}}{c}}\right]\right)}{2g}$$

**Problem 281: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \text{ArcCoth}[c x]) (d + e \text{Log}[f + g x^2])}{x^2} dx$$

Optimal (type 4, 560 leaves, 38 steps):

$$\begin{aligned} & \frac{2 a e \sqrt{g} \text{ArcTan}\left[\frac{\sqrt{g} x}{\sqrt{f}}\right]}{\sqrt{f}} - \frac{b e \sqrt{g} \text{ArcTan}\left[\frac{\sqrt{g} x}{\sqrt{f}}\right] \text{Log}\left[1 - \frac{1}{c x}\right]}{\sqrt{f}} + \\ & \frac{b e \sqrt{g} \text{ArcTan}\left[\frac{\sqrt{g} x}{\sqrt{f}}\right] \text{Log}\left[1 + \frac{1}{c x}\right]}{\sqrt{f}} + \frac{b e \sqrt{g} \text{ArcTan}\left[\frac{\sqrt{g} x}{\sqrt{f}}\right] \text{Log}\left[-\frac{2 \sqrt{f} \sqrt{g} (1-c x)}{(i c \sqrt{f}-\sqrt{g})(\sqrt{f}-i \sqrt{g} x)}\right]}{\sqrt{f}} - \\ & \frac{b e \sqrt{g} \text{ArcTan}\left[\frac{\sqrt{g} x}{\sqrt{f}}\right] \text{Log}\left[\frac{2 \sqrt{f} \sqrt{g} (1+c x)}{(i c \sqrt{f}+\sqrt{g})(\sqrt{f}-i \sqrt{g} x)}\right]}{\sqrt{f}} - \frac{(a + b \text{ArcCoth}[c x]) (d + e \text{Log}[f + g x^2])}{x} + \\ & \frac{1}{2} b c \text{Log}\left[-\frac{g x^2}{f}\right] (d + e \text{Log}[f + g x^2]) - \frac{1}{2} b c \text{Log}\left[\frac{g(1-c^2 x^2)}{c^2 f + g}\right] (d + e \text{Log}[f + g x^2]) - \\ & \frac{1}{2} b c e \text{PolyLog}\left[2, \frac{c^2 (f + g x^2)}{c^2 f + g}\right] + \frac{1}{2} b c e \text{PolyLog}\left[2, 1 + \frac{g x^2}{f}\right] - \\ & \frac{i b e \sqrt{g} \text{PolyLog}\left[2, 1 + \frac{2 \sqrt{f} \sqrt{g} (1-c x)}{(i c \sqrt{f}-\sqrt{g})(\sqrt{f}-i \sqrt{g} x)}\right]}{2 \sqrt{f}} + \frac{i b e \sqrt{g} \text{PolyLog}\left[2, 1 - \frac{2 \sqrt{f} \sqrt{g} (1+c x)}{(i c \sqrt{f}+\sqrt{g})(\sqrt{f}-i \sqrt{g} x)}\right]}{2 \sqrt{f}} \end{aligned}$$

Result (type 4, 1236 leaves):

$$\begin{aligned} & -\frac{a d}{x} - \frac{b d \text{ArcCoth}[c x]}{x} + b c d \text{Log}[x] - \\ & \frac{1}{2} b c d \text{Log}[1 - c^2 x^2] + a e \left( \frac{2 \sqrt{g} \text{ArcTan}\left[\frac{\sqrt{g} x}{\sqrt{f}}\right]}{\sqrt{f}} - \frac{\text{Log}[f + g x^2]}{x} \right) + \\ & \frac{1}{2} b e \left( -\frac{(2 \text{ArcCoth}[c x] + c x (-2 \text{Log}[x] + \text{Log}[1 - c^2 x^2])) \text{Log}[f + g x^2]}{x} - 2 c \left( \text{Log}[x] \right. \right. \\ & \left. \left. \left( \text{Log}\left[1 - \frac{i \sqrt{g} x}{\sqrt{f}}\right] + \text{Log}\left[1 + \frac{i \sqrt{g} x}{\sqrt{f}}\right] \right) + \text{PolyLog}\left[2, -\frac{i \sqrt{g} x}{\sqrt{f}}\right] + \text{PolyLog}\left[2, \frac{i \sqrt{g} x}{\sqrt{f}}\right] \right) \right) + \end{aligned}$$

$$\begin{aligned}
 & c \left( \text{Log} \left[ -\frac{1}{c} + x \right] \text{Log} \left[ \frac{c (\sqrt{f} - i \sqrt{g} x)}{c \sqrt{f} - i \sqrt{g}} \right] + \text{Log} \left[ \frac{1}{c} + x \right] \text{Log} \left[ \frac{c (\sqrt{f} - i \sqrt{g} x)}{c \sqrt{f} + i \sqrt{g}} \right] + \right. \\
 & \left. \text{Log} \left[ -\frac{1}{c} + x \right] \text{Log} \left[ \frac{c (\sqrt{f} + i \sqrt{g} x)}{c \sqrt{f} + i \sqrt{g}} \right] - \left( \text{Log} \left[ -\frac{1}{c} + x \right] + \text{Log} \left[ \frac{1}{c} + x \right] - \text{Log} [1 - c^2 x^2] \right) \right. \\
 & \left. \text{Log} [f + g x^2] + \text{Log} \left[ \frac{1}{c} + x \right] \text{Log} \left[ 1 - \frac{\sqrt{g} (1 + c x)}{i c \sqrt{f} + \sqrt{g}} \right] + \text{PolyLog} \left[ 2, \frac{c \sqrt{g} \left( \frac{1}{c} + x \right)}{i c \sqrt{f} + \sqrt{g}} \right] + \text{PolyLog} \left[ \right. \right. \\
 & \left. \left. 2, \frac{i \sqrt{g} (-1 + c x)}{c \sqrt{f} - i \sqrt{g}} \right] + \text{PolyLog} \left[ 2, -\frac{i \sqrt{g} (-1 + c x)}{c \sqrt{f} + i \sqrt{g}} \right] + \text{PolyLog} \left[ 2, \frac{i \sqrt{g} (1 + c x)}{c \sqrt{f} + i \sqrt{g}} \right] \right) - \\
 & \frac{1}{\sqrt{c^2 f g}} c g \left( 2 i \text{ArcCos} \left[ \frac{c^2 f - g}{c^2 f + g} \right] \text{ArcTan} \left[ \frac{c f}{\sqrt{c^2 f g} x} \right] - 4 \text{ArcCoth} [c x] \text{ArcTan} \left[ \frac{c g x}{\sqrt{c^2 f g}} \right] + \right. \\
 & \left( \text{ArcCos} \left[ \frac{c^2 f - g}{c^2 f + g} \right] + 2 \text{ArcTan} \left[ \frac{c f}{\sqrt{c^2 f g} x} \right] \right) \text{Log} \left[ \frac{2 g \left( c^2 f - i \sqrt{c^2 f g} \right) (-1 + c x)}{(c^2 f + g) \left( i \sqrt{c^2 f g} + c g x \right)} \right] + \\
 & \left( \text{ArcCos} \left[ \frac{c^2 f - g}{c^2 f + g} \right] - 2 \text{ArcTan} \left[ \frac{c f}{\sqrt{c^2 f g} x} \right] \right) \text{Log} \left[ \frac{2 g \left( c^2 f + i \sqrt{c^2 f g} \right) (1 + c x)}{(c^2 f + g) \left( i \sqrt{c^2 f g} + c g x \right)} \right] - \\
 & \left( \text{ArcCos} \left[ \frac{c^2 f - g}{c^2 f + g} \right] + 2 \left( \text{ArcTan} \left[ \frac{c f}{\sqrt{c^2 f g} x} \right] + \text{ArcTan} \left[ \frac{c g x}{\sqrt{c^2 f g}} \right] \right) \right) \\
 & \text{Log} \left[ \frac{\sqrt{2} e^{-\text{ArcCoth} [c x]} \sqrt{c^2 f g}}{\sqrt{c^2 f + g} \sqrt{-c^2 f + g + (c^2 f + g) \text{Cosh} [2 \text{ArcCoth} [c x]]}} \right] - \\
 & \left( \text{ArcCos} \left[ \frac{c^2 f - g}{c^2 f + g} \right] - 2 \left( \text{ArcTan} \left[ \frac{c f}{\sqrt{c^2 f g} x} \right] + \text{ArcTan} \left[ \frac{c g x}{\sqrt{c^2 f g}} \right] \right) \right) \\
 & \text{Log} \left[ \frac{\sqrt{2} e^{\text{ArcCoth} [c x]} \sqrt{c^2 f g}}{\sqrt{c^2 f + g} \sqrt{-c^2 f + g + (c^2 f + g) \text{Cosh} [2 \text{ArcCoth} [c x]]}} \right] + \\
 & i \left( \text{PolyLog} \left[ 2, \frac{\left( c^2 f - g - 2 i \sqrt{c^2 f g} \right) \left( \sqrt{c^2 f g} + i c g x \right)}{(c^2 f + g) \left( \sqrt{c^2 f g} - i c g x \right)} \right] - \right.
 \end{aligned}$$

$$\text{PolyLog}\left[2, \frac{\left(c^2 f - g + 2 i \sqrt{c^2 f g}\right) \left(\sqrt{c^2 f g} + i c g x\right)}{\left(c^2 f + g\right) \left(\sqrt{c^2 f g} - i c g x\right)}\right]\right)$$

**Problem 282: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + b \text{ArcCoth}[c x]) (d + e \text{Log}[f + g x^2])}{x^3} dx$$

Optimal (type 4, 712 leaves, 32 steps):

$$\begin{aligned} & \frac{b c e \sqrt{g} \text{ArcTan}\left[\frac{\sqrt{g} x}{\sqrt{f}}\right]}{\sqrt{f}} + \frac{a e g \text{Log}[x]}{f} + \frac{b e g \text{ArcCoth}[c x] \text{Log}\left[\frac{-2}{1+c x}\right]}{f} + \\ & b c^2 e \text{ArcTanh}[c x] \text{Log}\left[\frac{2}{1+c x}\right] - \frac{b e g \text{ArcCoth}[c x] \text{Log}\left[\frac{2 c (\sqrt{-f}-\sqrt{g} x)}{(c \sqrt{-f}-\sqrt{g})(1+c x)}\right]}{2 f} - \\ & \frac{1}{2} b c^2 e \text{ArcTanh}[c x] \text{Log}\left[\frac{2 c (\sqrt{-f}-\sqrt{g} x)}{(c \sqrt{-f}-\sqrt{g})(1+c x)}\right] - \frac{b e g \text{ArcCoth}[c x] \text{Log}\left[\frac{2 c (\sqrt{-f}+\sqrt{g} x)}{(c \sqrt{-f}+\sqrt{g})(1+c x)}\right]}{2 f} - \\ & \frac{1}{2} b c^2 e \text{ArcTanh}[c x] \text{Log}\left[\frac{2 c (\sqrt{-f}+\sqrt{g} x)}{(c \sqrt{-f}+\sqrt{g})(1+c x)}\right] - \frac{a e g \text{Log}[f + g x^2]}{2 f} - \\ & \frac{b c (d + e \text{Log}[f + g x^2])}{2 x} - \frac{(a + b \text{ArcCoth}[c x]) (d + e \text{Log}[f + g x^2])}{2 x^2} + \\ & \frac{1}{2} b c^2 \text{ArcTanh}[c x] (d + e \text{Log}[f + g x^2]) + \frac{b e g \text{PolyLog}\left[2, -\frac{1}{c x}\right]}{2 f} - \\ & \frac{b e g \text{PolyLog}\left[2, \frac{1}{c x}\right]}{2 f} - \frac{1}{2} b c^2 e \text{PolyLog}\left[2, 1 - \frac{2}{1+c x}\right] - \frac{b e g \text{PolyLog}\left[2, 1 - \frac{2}{1+c x}\right]}{2 f} + \\ & \frac{1}{4} b c^2 e \text{PolyLog}\left[2, 1 - \frac{2 c (\sqrt{-f}-\sqrt{g} x)}{(c \sqrt{-f}-\sqrt{g})(1+c x)}\right] + \frac{b e g \text{PolyLog}\left[2, 1 - \frac{2 c (\sqrt{-f}-\sqrt{g} x)}{(c \sqrt{-f}-\sqrt{g})(1+c x)}\right]}{4 f} + \\ & \frac{1}{4} b c^2 e \text{PolyLog}\left[2, 1 - \frac{2 c (\sqrt{-f}+\sqrt{g} x)}{(c \sqrt{-f}+\sqrt{g})(1+c x)}\right] + \frac{b e g \text{PolyLog}\left[2, 1 - \frac{2 c (\sqrt{-f}+\sqrt{g} x)}{(c \sqrt{-f}+\sqrt{g})(1+c x)}\right]}{4 f} \end{aligned}$$

Result (type 4, 1193 leaves):

$$\frac{1}{4 f x^2} \left( -2 a d f - 2 b c d f x - 2 b d f \text{ArcCoth}[c x] + 2 b c^2 d f x^2 \text{ArcCoth}[c x] + \right.$$

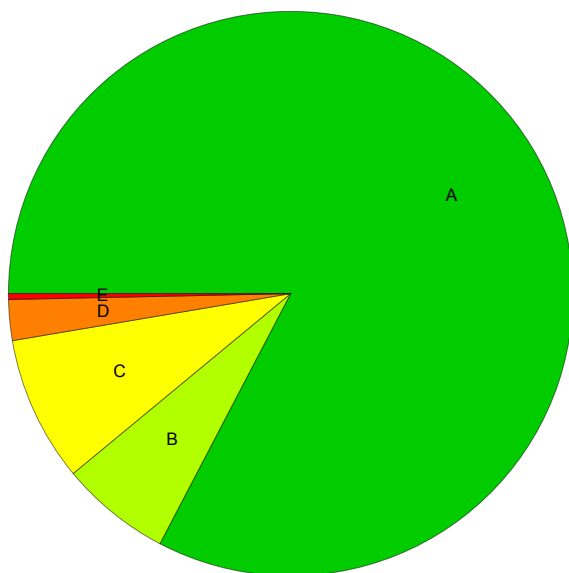
$$\begin{aligned}
 & 4 b c e \sqrt{f} \sqrt{g} x^2 \operatorname{ArcTan}\left[\frac{\sqrt{g} x}{\sqrt{f}}\right] + 4 i b c^2 e f x^2 \operatorname{ArcSin}\left[\sqrt{\frac{g}{c^2 f + g}}\right] \operatorname{ArcTanh}\left[\frac{c f}{\sqrt{-c^2 f g} x}\right] + \\
 & 4 i b e g x^2 \operatorname{ArcSin}\left[\sqrt{\frac{g}{c^2 f + g}}\right] \operatorname{ArcTanh}\left[\frac{c f}{\sqrt{-c^2 f g} x}\right] + \\
 & 4 b c^2 e f x^2 \operatorname{ArcCoth}[c x] \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcCoth}[c x]}\right] + 4 b e g x^2 \operatorname{ArcCoth}[c x] \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcCoth}[c x]}\right] - \\
 & 2 b c^2 e f x^2 \operatorname{ArcCoth}[c x] \operatorname{Log}\left[\frac{1}{c^2 f + g} e^{-2 \operatorname{ArcCoth}[c x]}\right. \\
 & \quad \left.\left(c^2 (-1 + e^{2 \operatorname{ArcCoth}[c x]}) f + g + e^{2 \operatorname{ArcCoth}[c x]} g - 2 \sqrt{-c^2 f g}\right)\right] - 2 b e g x^2 \operatorname{ArcCoth}[c x] \\
 & \operatorname{Log}\left[\frac{1}{c^2 f + g} e^{-2 \operatorname{ArcCoth}[c x]}\left(c^2 (-1 + e^{2 \operatorname{ArcCoth}[c x]}) f + g + e^{2 \operatorname{ArcCoth}[c x]} g - 2 \sqrt{-c^2 f g}\right)\right] + \\
 & 2 i b c^2 e f x^2 \operatorname{ArcSin}\left[\sqrt{\frac{g}{c^2 f + g}}\right] \operatorname{Log}\left[\frac{1}{c^2 f + g} e^{-2 \operatorname{ArcCoth}[c x]}\right. \\
 & \quad \left.\left(c^2 (-1 + e^{2 \operatorname{ArcCoth}[c x]}) f + g + e^{2 \operatorname{ArcCoth}[c x]} g - 2 \sqrt{-c^2 f g}\right)\right] + 2 i b e g x^2 \operatorname{ArcSin}\left[\sqrt{\frac{g}{c^2 f + g}}\right] \\
 & \operatorname{Log}\left[\frac{1}{c^2 f + g} e^{-2 \operatorname{ArcCoth}[c x]}\left(c^2 (-1 + e^{2 \operatorname{ArcCoth}[c x]}) f + g + e^{2 \operatorname{ArcCoth}[c x]} g - 2 \sqrt{-c^2 f g}\right)\right] - \\
 & 2 b c^2 e f x^2 \operatorname{ArcCoth}[c x] \operatorname{Log}\left[\frac{1}{c^2 f + g} e^{-2 \operatorname{ArcCoth}[c x]}\right. \\
 & \quad \left.\left(c^2 (-1 + e^{2 \operatorname{ArcCoth}[c x]}) f + g + e^{2 \operatorname{ArcCoth}[c x]} g + 2 \sqrt{-c^2 f g}\right)\right] - 2 b e g x^2 \operatorname{ArcCoth}[c x] \\
 & \operatorname{Log}\left[\frac{1}{c^2 f + g} e^{-2 \operatorname{ArcCoth}[c x]}\left(c^2 (-1 + e^{2 \operatorname{ArcCoth}[c x]}) f + g + e^{2 \operatorname{ArcCoth}[c x]} g + 2 \sqrt{-c^2 f g}\right)\right] - \\
 & 2 i b c^2 e f x^2 \operatorname{ArcSin}\left[\sqrt{\frac{g}{c^2 f + g}}\right] \operatorname{Log}\left[\frac{1}{c^2 f + g} e^{-2 \operatorname{ArcCoth}[c x]}\right. \\
 & \quad \left.\left(c^2 (-1 + e^{2 \operatorname{ArcCoth}[c x]}) f + g + e^{2 \operatorname{ArcCoth}[c x]} g + 2 \sqrt{-c^2 f g}\right)\right] - 2 i b e g x^2 \operatorname{ArcSin}\left[\sqrt{\frac{g}{c^2 f + g}}\right] \\
 & \operatorname{Log}\left[\frac{1}{c^2 f + g} e^{-2 \operatorname{ArcCoth}[c x]}\left(c^2 (-1 + e^{2 \operatorname{ArcCoth}[c x]}) f + g + e^{2 \operatorname{ArcCoth}[c x]} g + 2 \sqrt{-c^2 f g}\right)\right] + \\
 & 4 a e g x^2 \operatorname{Log}[x] - 2 a e f \operatorname{Log}[f + g x^2] - 2 b c e f x \operatorname{Log}[f + g x^2] - 2 a e g x^2 \operatorname{Log}[f + g x^2] - \\
 & 2 b e f \operatorname{ArcCoth}[c x] \operatorname{Log}[f + g x^2] + 2 b c^2 e f x^2 \operatorname{ArcCoth}[c x] \operatorname{Log}[f + g x^2] - \\
 & 2 b e g x^2 \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcCoth}[c x]}\right] - 2 b c^2 e f x^2 \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcCoth}[c x]}\right] + \\
 & b c^2 e f x^2 \operatorname{PolyLog}\left[2, \frac{e^{-2 \operatorname{ArcCoth}[c x]}\left(c^2 f - g + 2 \sqrt{-c^2 f g}\right)}{c^2 f + g}\right] + \\
 & b e g x^2 \operatorname{PolyLog}\left[2, \frac{e^{-2 \operatorname{ArcCoth}[c x]}\left(c^2 f - g + 2 \sqrt{-c^2 f g}\right)}{c^2 f + g}\right] + \\
 & b c^2 e f x^2 \operatorname{PolyLog}\left[2, -\frac{e^{-2 \operatorname{ArcCoth}[c x]}\left(-c^2 f + g + 2 \sqrt{-c^2 f g}\right)}{c^2 f + g}\right] +
 \end{aligned}$$

$$b e g x^2 \text{PolyLog}\left[2, -\frac{e^{-2 \text{ArcCoth}[c x]} \left(-c^2 f + g + 2 \sqrt{-c^2 f g}\right)}{c^2 f + g}\right]$$



## Summary of Integration Test Results

300 integration problems



A - 248 optimal antiderivatives

B - 19 more than twice size of optimal antiderivatives

C - 25 unnecessarily complex antiderivatives

D - 7 unable to integrate problems

E - 1 integration timeouts